Sisyphe v6.3 User's Manual

Pablo TASSI
Catherine VILLARET

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SISYPHE is the state of the art sediment transport and bed evolution module of the TELEMAC-MASCARET modelling system. SISYPHE can be used to model complex morphodynamics processes in diverse environments, such as coastal, rivers, lakes and estuaries, for different flow rates, sediment size classes and sediment transport modes.

In SISYPHE, sediment transport processes are grouped as bed-load, suspended-load or total-load, with an extensive library of bed-load transport relations. SISYPHE is applicable to non-cohesive sediments that can be uniform (single-sized) or non-uniform (multiple-sized), cohesive sediments (multi-layer consolidation models), as well as sand-mud mixtures. A number of physically-based processes are incorporated into SISYPHE, such as the influence of secondary currents to precisely capture the complex flow field induced by channel curvature, the effect of bed slope associated with the influence of gravity, bed roughness predictors, and areas of inerodible bed, among others.

For currents only, SISYPHE can be tightly coupled to the depth-averaged shallow water module TELEMAC-2D or to the three-dimensional Reynolds-averaged Navier-Stokes module TELEMAC-3D. In order to account for the effect of waves or combined waves and currents, SISYPHE can be internally coupled to the waves module TOMAWAC.

SISYPHE can be easily expanded and customized to particular requirements by modifying friendly, easy to read fortran files. To help the community of users and developers, SISYPHE includes a large number of examples, verification and validation tests for a range of applications.
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Executive Summary

The present document is part of the openTELEMAC documentation (http://www.opentelemac.org).
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Conventions

In this document, the following conventions are used:
– The names of the different modules are written in **caps and small caps**
– File names are written in **sans serif font**
– Keywords, variables, subroutines, etc. are written in **monospaced ‘typewriter’ font**
1 Introduction

Sisyphe is a sediment transport and morphodynamic model which is part of the hydroinformatics finite element and finite volume system Telemac-Mascaret. In Sisyphe, sediment transport rates, split into bedload and suspended load, are calculated at each node as a function of various flow (velocity, water depth, wave height, etc.) and sediment (grain diameter, relative density, settling velocity, etc.) parameters. The bedload is calculated by using a classical sediment transport formula from the literature. The suspended load is determined by solving an additional transport equation for the depth-averaged suspended sediment concentration. The bed evolution equation (Exner equation) can be solved by using either a finite element or a finite volume formulation.

Sisyphe is applicable to non-cohesive sediments (uniform or graded), cohesive sediments as well as sand-mud mixtures. The sediment composition is represented by a finite number of classes, each characterized by its mean diameter, grain density and settling velocity. Sediment transport processes can also include the effect of bottom slope, rigid beds, secondary currents and slope failure. For cohesive sediments, the effect of bed consolidation can be accounted for.

Sisyphe can be applied to a large variety of hydrodynamic flow conditions including rivers, estuaries and coastal applications. For the later, the effects of waves superimposed to a tidal current can be included. The bed shear stress, decomposed into skin friction and form drag, can be calculated either by imposing a friction coefficient (Strickler, Nikuradse, Manning, Chézy or user defined) or by a bed-roughness predictor.

In Sisyphe, the relevant hydrodynamic variables can be either imposed in the model (chaining method) or calculated by a hydrodynamic computation (internal coupling) by using one of the hydrodynamic modules of the Telemac system (modules Telemac-2D, Telemac-3D or Tomawac) or an external hydrodynamic model.

Sisyphe can be run on Unix, Linux or Windows. The latest release of Sisyphe (release 6.3) uses the version 6.3 of the Bief finite element library (Telemac system library). The reader can refer to the specific documentation for pre- and post-processing tools, e.g. [10].

Outline of the manual

This document is organized in three parts, as follows. In the first part (Part I), the main steps to run a sedimentological computation are given in Section 2. In Sections 3 and 4, a description of the main sediment and hydrodynamics parameters such that total bed shear stress, skin friction, etc. is presented. In the second part (Part II), specific aspects of non-cohesive sediment transport modelling are give. In Section 5 the bedload is presented. In Section 6, the suspended load transport is introduced. Details on bed evolution for bedload and suspended-load are given in Section 7. Section 8 presents the influence of the effect of waves on the sediment transport. In Section 9, sand grading effects are introduced. Finally, in Part III numerical aspects of cohesive and mixed (sand-mud) sediment transport modelling are introduced.
Part I
Basis
by Catherine Villaret & Pablo Tassi
2 Running a sedimentological computation

2.1 Input files

The minimum set of input files to run a Sisyph simulation includes the steering file (format ascii file cas), the geometry file (format selafin slf), and the boundary conditions file (format ascii file cli). Additional or optional input files include the fortran file, the reference file, the results file, etc.

2.2 Steering file

The steering file contains the necessary information for running a computation, including the physical and numerical parameters that are different from the default values. Like any other modules of the Telemac system, the model parameters can be specified in the (obligatory) Sisyph steering file. The following essential information (input/output) must be specified in the steering file:
- Input and output files
- Physical parameters (sand diameter, cohesive or not, settling velocity, etc.)
- Main sediment transport processes (transport mechanism and formulae, etc.)
- Additional sediment transport processes (secondary currents, slope effect, etc.)
- Numerical options and parameters (numerical scheme, options for solvers, etc.)

2.3 Coupling hydrodynamics and morphodynamics

We describe here two methods for linking the hydrodynamic and the morphodynamic models: by chaining (the flow is obtained from a previous hydrodynamic simulation assuming a fixed bed) or by internal coupling (both the flow and bed evolution are updated at each time step).

2.3.1 Chaining method

- Principle
  For this method, both models (hydrodynamic and morphodynamic) are running independently. During the first hydrodynamic simulation, the bed is assumed to be fixed. Then, in the subsequent morphodynamic step, the flow rate and free surface are read from the previous hydrodynamic results file. The chaining method is only justified for relatively simple flows, due to the difference in time-scales between the hydrodynamics and the bed evolution. For unsteady tidal flow, Sisyph can be used in unsteady mode, that means that the flow field is linearly interpolated between two time steps of the hydrodynamic file. For steady flows, the last time step of the hydrodynamic file is used and the flow rate and free surface is assumed to be constant while the bed evolves.

- Flow updating
  At each time step, the flow velocity is updated by assuming that both the flow rate and free surface elevation are conserved. In the case of deposition, the flow velocity is locally increased, whereas in the case of erosion, the flow velocity decreases. This schematic updating does not take into account any deviation of the flow. It is only suitable for simple flows and assuming relatively small bed evolutions. The morphodynamic computation is stopped when the bed evolution reaches a certain percent of the initial water depth. This methodology is not longer valid when the bed evolution becomes greater than a significant percentage of the water depth, specified by the user. At this point, it is recommended to stop the morphodynamic calculation and to recalculate the hydrodynamic variables.

- Mass continuity
  With this simple method, the sediment mass continuity may not be satisfied because of potential losses due to changes in the flow depth as the bed evolves. Furthermore, it can also be
responsible of numerical instabilities.

When the flow is steady (STEADY CASE = YES), only the last record of the previous result file will be used. Otherwise (STEADY CASE = NO), the TIDE PERIOD and NUMBER OF TIDES OR FLOODS will be used to specify the sequence to be read on the hydrodynamic file. Hydrodynamics records are interpolated at each time step of the sedimentological computation.

**Important**

An error may occur when the TIDE PERIOD is not a multiple of the graphic printout period of the hydrodynamic file (hydrodynamic file is not long enough). In an unsteady case, the keyword STARTING TIME OF THE HYDROGRAM gives the first time step to be read. If the starting time is not specified, the last period of the hydrogram will be used for the sedimentological computation.

**Steering and fortran files**

For uncoupled mode, the Sisyphe steering file should specify:

- The time steps, graphical or listing output, final time
- The hydrodynamic file as yielded by TELEMAC-2D or TELEMAC-3D (HYDRODYNAMIC FILE) or by the subroutine condim_sisyphe.f.
- For waves only: the wave parameters can be either calculated by a wave propagation code (TOMAWAC), or defined directly in Sisyphe (condim_sisyphe.f). The effect of waves on bed forms and associated bed roughness coefficient can be accounted with keyword: EFFECT OF WAVES = YES.
- A restart from a previous Sisyphe model run, by setting COMPUTATION CONTINUED = YES and specification of sedimentological results in PREVIOUS SEDIMENTOLOGICAL COMPUTATION
- Flow options: steady or unsteady options, flow period
- Friction options: Selects the type of formulation used for the bottom friction.

**Keywords**

For time step, final time and output:

- TIME STEP, NUMBER OF TIME STEPS
- GRAPHIC PRINTOUT PERIOD
- LISTING VARIABLES FOR GRAPHIC PRINTOUTS

For hydrodynamics (imposed flow and updated):

- HYDRODYNAMIC FILE
- STEADY CASE = NO, default option
- TIDE PERIOD = 44640, default option
- STARTING TIME OF THE HYDROGRAM = 0., default option
- NUMBER OF TIDES OR FLOODS = 1, default option
- CRITICAL EVOLUTION RATIO = 0.1, default value
- LAW OF BOTTOM FRICTION = 3, default value
- FRICTION COEFFICIENT = 50, default value

For waves:

- WAVE FILE, WAVE EFFECTS

**Important**

In the case of internal coupling, the friction coefficient is selected in the TELEMAC-2D or TELEMAC-3D steering file except when BED ROUGHNESS PREDICTION is set to YES.

### 2.3.2 Internal coupling

**Principle**

Sisyphe can be internally coupled to the hydrodynamic model, TELEMAC-2D or TELEMAC-3D. Sisyphe is internally coupled with the hydrodynamic model without any exchange of data files. The data to be exchanged between the two programs is now directly shared in the memory,
instead of being written and read in a file. At each time step, the hydrodynamics variables (velocity field, water depth and bed shear stress) are transferred into the morphodynamic model, which sends back the updated bed elevation to the hydrodynamic model.

- **Time step and coupling period**
  The internal coupling method is more time consuming than the chaining method. Various techniques can be set up to reduce the CPU time (e.g. parallel processors). In certain cases, the use of a coupling period > 1 allows the bed load transport rates and resulting bed evolution not to be re-calculated at every time step. For suspended load, a convection-diffusion equation needs to be solved. This transport equation obeys the same Courant number criteria on the time step than the hydrodynamics and therefore needs to be solved at each time step (COUPLING PERIOD = 1).
  The time step of Sisyphé is equal to the time step of Telemac-2d or Telemac-3d multiplied by the COUPLING PERIOD. The graphic and listing printout periods are the same as in the Telemac computation.
  The Telemac-2d and Telemac-3d steering files must specify the type of coupling, the name of the Sisyphé steering file, and the coupling period. In addition, the Fortran file of Sisyphé must be sometimes specified in the Telemac steering file (if for example there is no Fortran file for Telemac-2d or Telemac-3d steering file).

### Keywords

For internal coupling, the following keywords need to be specified in the Telemac-2d or Telemac-3d steering files:

- COUPLING WITH = SISYPHE
- COUPLING PERIOD =1, default value
- NAME OF SISYPHE STEERING FILE

All computational parameters (time step, duration, printout, option for friction) need to be specified in the Telemac-2d or Telemac-3d steering file, but are no longer used by Sisyphé. The values of time step, bottom shear stress, etc. are transferred directly to Sisyphé.

For long term simulations, it is possible to use the keyword MORPHOLOGICAL FACTOR (in order to speed-up the computational time. This method allows to increase the time step used for the morphological simulations. Further details can be found in [40].

### Keywords

For internal coupling, the following keyword need to be specified in the Sisyphé steering file:

- MORPHOLOGICAL FACTOR =1, default value

### Obsolete keywords

For the new version of Sisyphé, the following keywords are obsolete:

- TIME STEP
- GRAPHIC PRINTOUT PERIOD
- LISTING PRINTOUT PERIOD
- LAW OF BOTTOM FRICTION
- FRICTION COEFFICIENT
- VARIABLE TIME STEP (DTVAR)
- COMPUTATION METHOD (lengthening of the tide, replaced by MORPHOLOGICAL FACTOR
- FILTERING COEFFICIENT
- NON EQUILIBRIUM BED LOAD (NOEQBED)
- GRAIN FEEDING (LGRAFED)
2.4 Boundary conditions file

The format of the boundary condition file is the same as for TELEMAC-2D or TELEMAC-3D. This ascii file can be created by a mesh generator (for example, BlueKenue [10]) and modified using a text editor. Each line is related to a point along the edge of the mesh. Boundary points are listed in the file in the following way: First the domain outline points, proceeding counterclockwise from the lower left corner, then the islands proceeding clockwise, also from the lower left corner.

The edge points are numbered like the file lines; the numbering first describes the domain outline in the counterclockwise direction from lower left point (X+Y minimum), then the islands in the clockwise direction.

For TELEMAC-2D or TELEMAC-3D, the information contained in the hydrodynamics boundary condition file includes:

(1) (2) (3) (4) ... (8) (9)
LIHBOR LIUBOR LIVBOR QBOR ... LITBOR TBOR

Further details for the boundary condition file for Sisyphè are given in Chapter 5 (bedload) and Chapter 6 (suspended load).

2.5 Fortran files

Programming can be necessary for particular applications by modifying a user’s subroutine from the sources. When a subroutine(s) is necessary for a particular case, a Fortran file (keyword FORTRAN FILE) needs to be specified in the TELEMAC-2D or TELEMAC-3D steering file. Some typically used subroutines are briefly described below:

- **Coordinates modification**: `corrxy.f` was designed for carry-out, at the beginning of a computation, a modification of the nodes coordinates, e.g. for a scaling-up (shifting from a small-scale model to a full size), a rotation or a translation. This subroutine is empty by default and provides, in the form of a comment, an example of programmed change of scale and origin.

- **Time variable or space variable friction**: `strche.f` is used for defining a space-variable or time variable friction coefficient. By default, the value given for the FRICTION COEFFICIENT keyword in the steering file is imposed throughout the domain. The `corstr` subroutine is for imposing a time-variable friction coefficient, which can be computed as a function of liquid flow, water depth, etc.

- **Definition of rigid areas**: `noerod.f` is used for specifying the rigid areas. The depths of non-erodable areas (array ZR) are imposed in this subroutine.

- **New sand transport formula**: `qsform.f` can be used to program a sediment transport formula that is different from those already implemented in Sisyphè. The keyword BED-LOAD TRANSPORT FORMULA should be set to 0.

- **Declaration of private variables**: subroutines `nomvar.f` and `predes.f`. In the standard version of Sisyphè, it is possible to output a number of computed variables. In some cases, the user may want to compute variables and write them into the results file (so far, the number of these variables is limited to four). Likewise, each variable is identified by one or several letters in the keyword VARIABLES FOR GRAPHIC PRINTOUTS. The new variables are then identified by letters A, G, L and O which, respectively, correspond to numbers 23, 24, 25 and 26. In the data structure, these four variables correspond to the PRIVE%ADR(1)%P%X(X), PRIVE%ADR(2)%P%X(X), PRIVE%ADR(3)%P%X(X) and PRIVE%ADR(4)%P%X(X) arrays (where X is the number of grid points) that can be used in several programming stages. The new variables are programmed in two steps:

  - Firstly, the names of these new variables should be set by completing the `nomvar.f` subroutine, which consists of two equivalent structures for English and French languages. Each structure sets the names of variables of the results file to be generated, then the names of variables to be read again in the previous computation file in case of a reactivation. This su-
broutine can also be amended when, for instance, the French version of Sisyphe being used, one wants to make a reactivation on a file generated by the English version. If so, the TEXTPR array of the subroutine French part shall contain the English names of the variables.

- Secondly, the PREDES subroutine is to be amended for entering the computation(s) of the new variable(s) thereinto. Besides, the LEO and IMP variables are provided for determining whether, at the time step being considered, the variable should be printed in the monitoring listing or in the results file.

**Important**

In the case of internal coupling, the Sisyphe fortran file can be specified in the Telemac-2d or Telemac-3d steering file instead. This is necessary if the fortran file is not present for the hydrodynamics.

### 2.6 Parallel computing

For running simulations in parallel mode, the user must specify the keyword PARALLEL PROCESSORS in the Telemac-2d or Telemac-3d steering file (for internal coupling simulations).
3 Hydrodynamics parameters

3.1 Current induced bed shear stresses

3.1.1 Coupled mode

The bed shear stress is one of the most important hydrodynamics parameters regarding sediment transport applications. The current-generated bed shear stress is used in 2D in the shallow water momentum equation and in 3D as the bottom boundary condition for the velocity profile. When Sisyphe is internally coupled with Telemac-2d, the bed shear stress term \( \tau_0 = (\tau_x, \tau_y) \) is calculated at each time step from the classical quadratic dependency on the depth-averaged velocity:

\[
\tau_{x,y} = \frac{1}{2} \rho C_d (u, v) |u|, \tag{1}
\]

where \((u, v)\) are the depth-averaged velocity components along the \(x\)– and \(y\)– Cartesian directions, respectively; \(|u| = |(u, v)| = \sqrt{u^2 + v^2}\) the velocity module; and friction coefficient \(C_d\). The magnitude of the bed shear stress \(\tau_0\) can be related to the friction velocity \(u_*\), defined by:

\[
\tau_0 = \rho u_*^2. \tag{2}
\]

### Sisyphe coupled with Telemac-2d

- If the model is coupled with Telemac-2d, the flow variables and the values of the friction coefficients (and therefore, the bed shear stresses) are provided by Telemac-2d. The bed shear stress and resulting bedload transport rates are assumed to be in the direction of the mean flow velocity, except when the sediment transport formulation accounts for:
  - deviation correction (bed slope effect), see § 5.3
  - secondary currents, see § 5.5

### Sisyphe coupled with Telemac-3d

When Sisyphe is coupled with Telemac-3d, the bed shear stress is aligned with the near bed velocity in order to account for possible flow deviations. The magnitude of the bed shear stress is still related to the depth-averaged velocity, except if the Nikuradse friction law is applied. In this case, the friction velocity is related to the near bed flow velocity \(u(z_1)\) by assuming a logarithmic velocity profile near the bed:

\[
u(z_1) = \frac{u_*}{k} \ln \left( \frac{z_1}{z_0} \right) \tag{3}
\]

where \(z_0\) is expressed as a function of the Nikuradse bed roughness \((z_0 = k_s / 30)\), with \(k_s\) the grain roughness height, \(z_1\) is the distance of the first vertical plane from the bed level; and \(k = 0.4\) is the von Kármán constant. For flat beds, the roughness height has been shown to be approximately \(k_s \approx 3d_{50}\) [29], with \(d_{50}\) the grain size with 50% of the material finer by weight.

3.1.2 Uncoupled mode

The quadratic friction coefficient \(C_d\), which is used to calculate the total bed shear stress, can be calculated after the selected friction law. Different options, which are consistent with Telemac-2d options are available in Sisyphe and depend on the choice of the keywords LAW FOR BOTTOM FRICTION and on the value of the FRICTION COEFFICIENT. Similar keywords are available in order to account for the effect of the friction for Telemac-2d, Telemac-3d and Sisyphe.
- Chézy coefficient $C_h$ ($\text{KFR0T} = 2$)

$$C_d = \frac{2g}{C_h^2}$$  \hspace{1cm} (4)

- Strickler coefficient $S_t$ ($\text{KFR0T} = 3$)

$$C_d = \frac{2g}{S_t^2 h^{1/3}}$$  \hspace{1cm} (5)

- Manning friction $M_a$ ($\text{KFR0T} = 4$)

$$C_d = \frac{2g}{h^{1/3} M_a^2}$$  \hspace{1cm} (6)

- Nikuradse bed roughness $k_s$ ($\text{KFR0T} = 5$)

$$C_d = 2 \left[ \frac{\kappa}{\log \left( \frac{12k_s}{h} \right)} \right]^2,$$  \hspace{1cm} (7)

**Keywords**
In the Sisyphe steering file, for the uncoupled mode the total bed shear stress is calculated based on:
- LAW OF BOTTOM FRICTION ($\text{KFR0T} = 3$, default option)
- BOTTOM FRICTION COEFFICIENT ($S_t = 50$, default value)

**Important**
In the case of internal coupling with TELEMAC-2d or TELEMAC-3d, the friction keywords provided in the Sisyphe steering file are not longer used.

### 3.2 Bed roughness

#### 3.2.1 Role of bed forms

A natural sediment bed is generally covered with bed forms, in general characterized by their length $\lambda_d$ and height $\eta_d$. The presence of bed forms modifies the boundary layer flow structure, with the formation of recirculation cells and depressions in the lee of bed forms. Depending on the flow and sediment transport rates, the size of bed forms ranges from a few centimeters for ripples to tens of meters for dunes or bars. The dimension of dunes scales with the water depth $h$, such that $\eta_d \approx 0.4h$ and $\lambda_d \approx 6 - 10h$. In most cases, large scale models do not resolve the small to medium scale bed forms (ripples, mega-ripples) which need therefore to be parameterized by increasing the friction coefficient. To determine bed roughness, there are two options available in Sisyphe. The simplest one is to impose the friction coefficient based on friction laws. As in most applications the bed roughness is unknown, an alternative is to predict the value of the bed roughness as a function of flow and sediment parameters using a bed roughness predictor.

**Important**
In coupled mode, the bed roughness predicted by Sisyphe is sent to the hydrodynamics model in order to avoid inconsistencies between hydrodynamics and morphodynamics simulations.

#### 3.2.2 Bed roughness predictor

Different options are programmed in Sisyphe to predict the total bed roughness through the associated keywords BED ROUGHNESS PREDICTION and BED ROUGHNESS PREDICTOR OPTION. It is recalled
that the bed friction option of Sisyphé is not used in the case of internal coupling with Telemac-2D or Telemac-3D.

If the keyword BED ROUGHNESS PREDICTION is activated (= YES), the following options are available via the keyword BED ROUGHNESS PREDICTOR OPTION:

- For IKS = 1: the bed is assumed to be flat \( k_s = k'_s = a d_{50} \), with \( a \) a constant (assumed to be equal to 3). The \( a \) parameter can be changed according to the keyword RATIO BETWEEN SKIN FRICTION AND MEAN DIAMETER.
- IKS = 2: the bed is assumed to be covered by ripples.
- For currents only, the ripple bed roughness is function of the mobility number \( \Psi = U^2/(s - 1)g d_{50} \), see [54]:
  \[
  k_r = \begin{cases} \frac{d_{50}(85 - 65 \tanh(0.015(\Psi - 150)))}{20d_{50}} & \text{for } \Psi < 250 \\ 0 & \text{otherwise} \end{cases}
  \]

- For waves and combined waves and currents, bedform dimensions are calculated as a function of wave parameters following the method of Wiberg and Harris [60]. The wave-induced bedform bed roughness \( k_r \) is calculated as a function of the wave-induced bedform height \( \eta_r \):
  \[
  k_r = \max(k'_s, \eta_r).
  \]

Then,
\[
k_s = k'_s + k_r.
\]

- IKS = 3: for currents only, the van Rijn’s total bed roughness predictor has been implemented [27, 54]. The total bed roughness can be decomposed into a grain roughness \( k'_s \), a small-scale ripple roughness \( k_r \), a mega-ripple component \( k_{mr} \), and a dune roughness \( k_d \):
  \[
  k_s = k'_s + \sqrt{k^2_r + k^2_{mr} + k^2_d}.
  \]

Both small scale ripples and grain roughness have an influence on the sediment transport laws, while the mega-ripples and dune roughness only contribute to the hydrodynamic model (total friction). In Equation (10), the general expression for megaripple roughness \( k_{mr} \) is given by:

\[
k_{mr} = 0.00002f_{fs}h(1 - \exp^{-0.05\Psi})(550 - \Psi),
\]

with
\[
f_{fs} = \begin{cases} \frac{d_{50}}{(1.5d_{sand})} & \text{for } d_{50} < 1.5d_{sand} \\ 1 & \text{otherwise} \end{cases}
\]

and the general expression for dune roughness \( k_d \) is estimated by:

\[
k_d = 0.00008f_{fs}h(1 - \exp^{-0.02\Psi})(600 - \Psi).
\]

Keywords
In Sisyphé steering file, the bed roughness prediction is calculated based on:

- BED ROUGHNESS PREDICTION, (= NO default option)
- BED ROUGHNESS PREDICTOR OPTION (IKS = 1: Flat bed (default option), IKS = 2: Rippled bed, IKS = 3: Dunes and mega ripples (van Rijn’s method))
- RATIO BETWEEN SKIN FRICTION AND MEAN DIAMETER = 3, default option. If = 0 use skin friction prediction from Van Rijn (2007) for currents or the Wiberg and Harris method for waves.
3.3 Skin friction

3.3.1 Bed shear stress decomposition

In the presence of bedforms, the total bed shear stress can be expressed as the sum of two components:

$$\tau_0 = \tau_0' + \tau_0''$$  \hspace{1cm} (11)

where $\tau_0$ is the total bed shear stress, $\tau_0'$ is the grain (or skin) shear stress, and $\tau_0''$ is the form shear stress. The local skin friction component determines the bedload transport rate and the equilibrium concentration for the suspension. The total friction velocity determines the turbulence eddy viscosity/diffusivity vertical distribution in 3D models, and therefore determines both the velocity vertical profile and the mean concentration profile.

Bedload transport rates are calculated as a function of the local skin friction component $\tau_0'$. The total bed shear stress issued from the hydrodynamics model needs to be corrected in the morphodynamics model as follows:

$$\tau_0' = \mu \tau_0$$  \hspace{1cm} (12)

where $\mu$ is a correction factor for skin friction. Physically, the skin bed roughness should be smaller than the total bed roughness (i.e. $\mu \leq 1$). However, in most cases the hydrodynamic friction does not represent the physical bottom friction: the coefficient $\mu$ is generally used as a calibration coefficient in hydrodynamics models. It is adjusted by comparing simulation results with observations of the time-varying free surface and velocity field. Therefore, its model value integrates various neglected processes (side wall friction, possible errors in the bathymetry and input data). Under those conditions, a correction factor $\mu > 1$ can be admitted.

3.3.2 Correction factor for skin friction

Different methods are programmed in Sisyphe in order to calculate the bedform correction factor $\mu$, according to the keyword SKIN FRICTION CORRECTION:

- $ICR = 0$: no correction, the total friction issued from Telemac is directly used for sand transport calculations ($\mu = 1$).
- $ICR = 1$: the skin roughness is assumed to be proportional to the sand grain diameter like in the case of flat beds ($k'_s \sim d_{50}$). The proportionality coefficient is specified by the keyword RATIO BETWEEN SKIN FRICTION AND MEAN DIAMETER, defined as:

$$\mu = \frac{C'_d}{C_d}$$  \hspace{1cm} (13)

where $C_d$ et $C'_d$ are both quadratic friction coefficients related to total friction and skin friction, respectively. $C_d$ is obtained from Telemac-2d or Telemac-3d and $C'_d$ is calculated from $k'_s$, as follows:

$$C'_d = 2 \left[ \frac{k}{\log \left( \frac{12h}{k'_s} \right)} \right]^2$$  \hspace{1cm} (14)

- $ICR = 2$ the bedform predictor is used to calculate the bedform roughness $k_r$ in order to account for the effect of ripples. Both $k_r$ and $k'_s$ should influence the transport rates. It is assumed that:

$$\mu = \frac{C_d^{0.75} C'_d^{0.25}}{C_d}$$  \hspace{1cm} (15)

where the quadratic friction $C_r$ due to bedforms is calculated as a function of $k_r$. 

Keywords

The option selected for the skin friction correction is based on keywords:

- SKIN FRICTION CORRECTION (ICR=0, default option)
4 Sediment parameters

4.1 Sediment properties

Fine sediment particles of grain size $d_{50} < 60\mu m$ present complex cohesive properties which affect the sediment transport processes. For non-cohesive sediments (median diameter $d_{50} > 60\mu m$), the grain diameter and grain density $\rho_s$ are the key parameters which determine its resistance to erosion and sediment transport rate.

In Part II, we consider non-cohesive sediments either characterized by one single value for the grain size $d_{50}$ and grain density $\rho_s$ or represented by a number of classes for sand grading.

In Part III, we deal with cohesive sediments ($d_{50} < 60\mu m$) : the grain diameter is no longer the key sediment parameter and the settling velocity now depends on the flocculation state of suspension, whereas the critical bed shear strength depends on the consolidation state of the sediment bed.

### Keywords

The physical properties of the sediment are always defined in the Sisyphe steering file using the following keywords :

- COHESIVE SEDIMENTS (= NO, default option)
- NUMBER OF SIZE-CLASSES OF BED MATERIAL (NSICLA= 1, default option)
- SEDIMENT DIAMETERS ($d_{50} > 60\mu m$ for non-cohesive sediments)
- SEDIMENT DENSITY ($\rho_s = 2650 \text{ kgm}^{-3}$, default value)

4.2 Shields parameter

The critical Shields number or dimensionless critical shear stress $\Theta_c$ is defined by :

$$\Theta_c = \frac{\tau_c}{g(\rho_s - \rho)d_{50}}$$  \hspace{1cm} (16)

where $\tau_c$ is the critical shear stress for sediment incipient motion. Values of $\Theta_c$ can be either specified in the Sisyphe steering file by use of keyword SHIELDS PARAMETERS or calculated by the model as a function of non-dimensional grain diameter $D_s$ :

$$D_s = d([(\rho_s/\rho - 1)g/\nu^2]^{1/3}.$$  \hspace{1cm} (17)

The critical Shields number is implemented in the subroutine init_sediment.f as follows :

$$\Theta_c = \begin{cases} 0.24D_s^{-1}, & D_s \leq 4 \\ 0.14D_s^{-0.64}, & 4 < D_s \leq 10 \\ 0.04D_s^{-0.10}, & 10 < D_s \leq 20 \\ 0.013D_s^{0.29}, & 20 < D_s \leq 150 \\ 0.045, & 150 \leq D_s \\ \end{cases}$$

where $\tau_c$ and $d_{50}$ are in N m$^{-2}$ and in m, respectively. If the value of the Shields number is not specified in the steering file, Sisyphe computes the value according the Equation (4.2) as a function of the non-dimensional grain diameter $D_s$. For multi-grain sizes, the Shields parameter needs to be specified for each class.

### Keywords

- SHIELDS PARAMETERS
4.3 **Settling velocities**

The settling velocity $w_s$ is an important parameter for suspended sediment transport. It can be either specified or calculated by the model as a function of the grain diameter. The van Rijn formula [52, 53] which is valid for non-cohesive spherical particles and dilute suspensions, has been implemented in **Sisyph** as follows:

$$w_s = \begin{cases} 
\frac{(s - 1)gd_{50}^2}{18v}, & \text{if } d_{50} \leq 10^{-4} \\
\frac{10v}{d_{50}} \left(1 + 0.01 \frac{(s - 1)gd_{50}^2}{18v^2} - 1\right), & \text{if } 10^{-4} \leq d_{50} \leq 10^{-3} \\
1.1 \sqrt{(s - 1)gd_{50}}, & \text{otherwise}
\end{cases}$$

with $s = \rho_s / \rho_0$ is the relative density, $v$ is the fluid viscosity and $g$ is gravity.

**Keywords**

- SETTLING VELOCITIES

**Attention**

The default value is not given. If the user does not give a value, the subroutine `vichu.f` computes their value as a function of the grain size, the relative density and the fluid viscosity.

4.4 **Transport rate**

When the current-induced bed shear stress exceeds the critical threshold value coarse sediment particles start to move as bedload, while the finer particles are transported in suspension. The total sediment load $Q_t$ includes a bedload $Q_b$ and suspended load $Q_s$ components:

$$Q_t = Q_b + Q_s. \quad (18)$$

Bedload occurs in a very thin high concentrated near-bed layer, where inter-particle interactions develop. In **Sisyph**, we assume the classical delimitation between bedload and suspended load. The interface between the bedload and suspended load is located at $z = Z_{ref}$:

- In the thin high-concentrated bedload layer ($z < Z_{ref}$) inter-particle interactions and flow-turbulent interactions strongly modify the flow structure. Equilibrium conditions are however a reliable assumption to relate the bed-load to the current induced bed shear stress.
- In the upper part of the flow ($z > Z_{ref}$), for dilute suspension clear flow concepts still apply, and the sediment grains can be regarded as a passive scalar which follows the mean and turbulent flow velocity, with an additional settling velocity term.
Part II
Non-cohesive sediments
by Catherine Villaret & Pablo Tassi
5 Bedload transport

5.1 Bedload transport formulae

For currents only (no wave effects), a large number of semi-empirical formulae can be found in the literature to calculate the bedload transport rate. **Sisyphe** offers the choice among different bedload formulae including the Meyer-Peter and Müller, Engelund-Hansen and Einstein-Brown formulae.

Most sediment transport formulae assume threshold conditions for the onset of erosion (e.g. Meyer-Peter and Müller, van Rijn and Hunziker). Other formulae are based on similar energy concept (e.g. Engelund-Hansen) or can be derived from a statistical approach (e.g. Einstein-Brown, Bijker, etc.). The non-dimensional current-induced sand transport rate \( \Phi_s \), is expressed as:

\[
\Phi_s = \frac{Q_b}{\sqrt{g(s-1)d_{ch}^3}}
\]

with \( \rho_s \) the sediment density; \( s = \rho_s/\rho \) the relative density; \( d \) the characteristic sand grain diameter (= \( d_{ch} \) for uniform grains); and \( g \) the gravity. The characteristic sand grain diameter can be chosen as \( d_{50} \) initially. As presented next, the non-dimensional sand transport rate \( \Phi_s \) is, in general, expressed as a function of the non-dimensional skin friction or Shields parameter \( \theta' \), defined by:

\[
\theta' = \frac{\mu \tau_0}{(\rho_s - \rho)gd_{ch}}
\]

with the correction factor for skin friction \( \mu \) and the bottom shear stress \( \tau_0 \).

5.2 Sediment transport formulae

The choice of a transport formula depends on the selected value of the model parameter ICF, as defined in the steering file by the keyword **BED-LOAD TRANSPORT FORMULA**. By setting \( \text{ICF} = 0 \), the user can program a specific transport formula through the subroutine **qsform.f**.

### Keywords

- **BED-LOAD TRANSPORT FORMULA** (default option : Meyer-Peter-Müller formula \( \text{ICF}=1 \))

- \( \text{ICF} = 1 \) (Meyer-Peter-Müller formula) : this classical bed-load formula has been validated for coarse sediments in the range \( 0.4 \text{ mm} < d_{50} < 29 \text{ mm} \). It is based on the concept of initial entrainment:

\[
\Phi_b = \begin{cases} 
0 & \text{if } \theta' < \theta_c \\
\alpha_{mpm}(\theta' - \theta_c)^{3/2} & \text{otherwise}
\end{cases}
\]

with \( \alpha_{mpm} \) a coefficient (= 8 by default), \( \theta_c \) the critical Shields parameter (= 0.047 by default). The coefficient \( \alpha_{mpm} \) can be modified in the steering file by the keyword **MPM COEFFICIENT** (= 8.0 by default). The characteristic sand grain diameter was originally chosen to \( d_{64} \) by Meyer-Peter and Müller.

- \( \text{ICF} = 2 \) (Einstein-Brown formula) : this bed-load formula is recommended for gravel \( (d_{50} > 2 \text{ mm}) \) and large bed shear stress \( \theta > \theta_c \). The solid transport rate (see [17]) is expressed as:

\[
\Phi_b = F(D_*)f(\theta'),
\]

with

\[
F(D_*) = \left( \frac{2}{3} + \frac{36}{D_*} \right)^{0.5} - \left( \frac{36}{D_*} \right)^{0.5},
\]

where

\[
D_* = \frac{g}{\mu \tau_0}.
\]
and
\[ f(\theta') = \begin{cases} 
2.15 \exp\left(-0.391/\theta'\right) & \text{if } \theta' \leq 0.2 \\
40 \theta'^3 & \text{otherwise}
\end{cases} \]

where the non-dimensional diameter \( D_s \) is defined according to Equation (17).

- ICF = 3 or 30 (Engelund-Hansen formula): this formula predicts the total load (bedload + suspended load). It is recommended for fine sediments, in the range \( 0.2 \text{ mm} < d_{50} < 1 \text{ mm} \) but beware that the use of a total load formula is only suitable under equilibrium conditions (quasi-steady and uniform flow). The two different forms of the same equation are programmed in SISYPHE:
  - ICF = 30 corresponds to the original formula, where the transport rate is related to the skin friction without threshold:
    \[ \Phi_b = 0.1 \theta'^{5/2} \]  
    (23)
  - ICF = 3 corresponds to the version modified by Chollet and Cunge [12] to account for the effects of sand dunes. The transport rate is related to the total bed shear stress as:
    \[ \Phi_s = \frac{0.1}{C_d} \hat{\theta}^{5/2}, \]  
    (24)

where the dimensionless bed shear stress \( \hat{\theta} \) is calculated as a function of the dimensionless skin friction \( \theta' \):
\[ \hat{\theta} = \begin{cases} 
0 & \text{if } \theta' < 0.06 \text{ (flat bed regime - no transport)} \\
\sqrt{2.5(\theta' - 0.06)} & \text{if } 0.06 < \theta' < 0.384 \text{ (dune regime)} \\
1.065 \theta'^{0.176} & \text{if } 0.384 < \theta' < 1.08 \text{ (transition regime)} \\
\theta' & \text{if } \theta' > 1.08 \text{ (flat bed regime - upper regime)}
\end{cases} \]

\[ \text{Attention} \]
To avoid the suspended load to be calculated twice, in the case of coupling between bedload and suspended load (SUSPENSION = YES), the total load formula (ICF = 3 or 30) should not be used.

- ICF = 7 (van Rijn’s formula): this formula was proposed by van Rijn [51] to calculate the bedload transport rate for particles of size \( 0.2 \text{ mm} < d_{50} < 2 \text{ mm} \):
\[ \Phi_b = 0.053 D_s^{-0.3} \left( \frac{\theta_p - \theta_{cr}}{\theta_{cr}} \right)^{2.1}. \]  
    (25)

\[ \text{Attention} \]
Other sediment transport formulae, described in Chapters 8 and 9, have been programmed in SISYPHE to account for the effects of waves (cf. Bijker, Bailard, Dibajnia and Watanabe, etc.) or sand grading (cf. Hunziker). The Bijker’s formula (ICF = 4) is a total load formula (bedload + suspended load) that can also be used to account for currents only.

\subsection{5.2.1 Validity range of sediment transport formulae}

Most sediment transport formulae are based on experiments performed under fluvial, unidirectional flows. These formulae shown a rapid variation of the bedload transport prediction, as a function of the mean flow intensity. Therefore, an increasing of the current velocity by 10% will result, depending on the formula being used, in an increasing of the transport rate of over 30% (Meyer-Peter), 60% (Engelund-Hansen) or almost 80% (Einstein-Brown). Therefore, any error made when calculating the hydrodynamics will be significantly amplified by the sediment transport rates estimates. On the other hand, under variable flow conditions (e.g. tidal regime), the average transport will be highly influenced by the stronger currents and will not be directly related to the mean flow.
The validity range of the different formulae is briefly summarized in Table 1.

<table>
<thead>
<tr>
<th>Formula</th>
<th>Meyer-Peter&amp; Müller</th>
<th>Einstein-Brown</th>
<th>Engelund-Hansen</th>
<th>van Rijn</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFC</td>
<td>1</td>
<td>2</td>
<td>3 or 30</td>
<td>7</td>
</tr>
<tr>
<td>Mode of transport</td>
<td>bedload</td>
<td>bedload</td>
<td>total load</td>
<td>bedload</td>
</tr>
<tr>
<td>Validity range ((d_{50}))</td>
<td>0.4 – 29 mm</td>
<td>0.25 – 32 mm</td>
<td>0.19 – 0.93 mm</td>
<td>0.20 – 2.0 mm</td>
</tr>
</tbody>
</table>

Table 1 – Validity range of some of the sand transport formulae programmed in Sisyphe (for currents only).

5.3 Bed slope effect

The effect of a sloping bottom is to increase the bedload transport rate in the downslope direction, and to reduce it in the upslope bedload direction. In Sisyphe, a correction factor can be applied to both the magnitude and direction of the solid transport rate, before solving the bed evolution equation. The bed slope effect is activated if the keyword SLOPE EFFECT is present in the Sisyphe steering file.

Two different formulations for both effects are available depending on the choice of the keywords FORMULA FOR SLOPE EFFECT, which chooses the magnitude correction and FORMULA FOR DEVIATION, which chooses the direction correction. At the open boundaries the slope effect is disarmed due to stability reasons.

5.3.1 Correction of the magnitude of bedload transport rate (formula for slope effect)

- SLOPEFF = 1: this correction method is based on the Koch and Flokstra’s formula [31]. The solid transport rate intensity \( Q_b \) is multiplied by a factor \( 1 - \beta \frac{\partial Z_f}{\partial s} \), thus:

\[
Q_b = Q_{b0} \left( 1 - \beta \frac{\partial Z_f}{\partial s} \right), \tag{26}
\]

with \( s \) the coordinate in the current direction and \( \beta \) an empirical factor, which can be specified with the associated keyword BETA (default value = 1.3). This effect of bed slope is then similar to adding a diffusion term in the bed evolution equation. It tends to smooth the results and is often used to reduce instabilities.

- SLOPEFF=2: this correction is based on the method of Soulsby (1997), in which the threshold bed shear stress \( \theta_{co} \), modified as a function of the bed slope \( \chi \), the angle of repose of the sediment \( \phi_s \), modified in the Sisyphe steering file with the keyword FRICTION ANGLE OF THE SEDIMENT (= 40° by default), and the angle of the current to the upslope direction \( \psi \):

\[
\frac{\theta_c}{\theta_{co}} = \frac{\cos \psi \sin \chi + (\cos^2 \chi \tan^2 \phi_s - \sin^2 \psi \sin^2 \chi)^{0.5}}{\tan \phi_s}, \tag{27}
\]

with \( \theta_c \) the modified threshold bed shear stress. Be aware that this formula effected the results with threshold bedload formulae (like Meyer-Peter and Müller) only.

5.3.2 Correction of the direction of bedload transport rate (formula for deviation)

The change in the direction of solid transport is taken into account by the formula:

\[
\tan \alpha = \tan \delta - T \frac{\partial Z_f}{\partial n}, \tag{28}
\]

where \( \alpha \) is the direction of solid transport in relation to the flow direction, \( \delta \) is the direction of bottom stress in relation to the flow direction, and \( n \) is the coordinate along the axis perpendicular to the flow.
DEVIA = 1: according to Koch and Flochstra, the coefficient $T$ depends exclusively on the Shields parameter

$$T = \frac{4}{6\theta}$$  \hfill (29)

DEVIA = 2: the deviation is calculated based on Talmon et al. [44] and depends on the Shields parameter and an empirical coefficient $\beta_2$

$$T = \frac{1}{\beta_2 \sqrt{\theta}}$$ \hfill (30)

The empirical coefficient $\beta_2$ can be modified by the keyword PARAMETER FOR DEVIATION ($= 0.85$ by default). For rivers could be achieved satisfying results by using a higher value of $\beta_2 = 1.5$.

### Keywords

- SLOPE EFFECT (= NO, default option)
- FORMULA FOR SLOPE EFFECT (SLOPEFF = 1, default option)
- FORMULA FOR DEVIATION (DEVIA = 1, default option)
- FRICTION ANGLE OF THE SEDIMENT ($\phi_s = 40^\circ$, default value)
- BETA ($\beta = 1.3$, default value)
- PARAMETER FOR DEVIATION ($\beta_2 = 0.85$, default value)

---

**Figure 1** – Sloping bed: general slope (adapted from [42]).

### 5.4 Sediment Slide

An iterative algorithm prevents the bed slope to become greater than the maximum friction angle ($\theta_s \approx 32^\circ - 40^\circ$). This option is activated by use of key word SEDIMENT SLIDE. Further details can be found in the Die-Moran’s thesis [15].

### Keywords

- SEDIMENT SLIDE = NO, default value
- FRICTION ANGLE OF SEDIMENT = 40°, default value
5.5 Bedload transport in curved channels

The bedload movement direction deviates from the main flow direction due to helical flow effect [65]. Different authors proposed empirical formulas for evaluating this deviation. The Engelund formula [19], based on the assumption that the bottom shear stress, the bed roughness and the mean water depth are constant in the cross-section, is

\[
\tan \delta = \frac{7h}{r} \quad (31)
\]

where \( \delta \) is the angle between the bedload movement and main flow direction, \( h \) the mean water depth and \( r \) the local radius of curvature. Yalin and Ferrera da Silva [64] have showed that \( \delta \) is proportional to \( h/r \). The local radius \( r \) can be computed based on the the cross sectional variation of the free surface [19]:

\[
r = -\rho a' \frac{U^2}{g \frac{\partial Z}{\partial y}},
\]

with \( a' \) a coefficient (\( a' \approx 0.75 \) for very rough beds and \( a' \approx 1.0 \) for smooth beds).

**Keywords**
- SECONDARY CURRENTS (= NO, default option)
- SECONDARY CURRENTS ALPHA COEFFICIENT (= 1.0, default option)

**Attention**
- This option should only be activated when the flow is calculated by a depth-averaged model.
6 Suspended load

6.1 Main assumptions

6.1.1 Vertical concentration profile

We assume a Rouse profile for the vertical concentration distribution, which is theoretically valid in uniform steady flow conditions:

\[ C(z) = C_{ref} \left( \frac{z - h}{z} \frac{a}{a + h} \right)^R, \]

where \( R \) is the Rouse number defined by:

\[ R = \frac{w_s}{\kappa u_*}, \]

with \( w_s > 0 \) the vertical-settling sediment velocity, \( \kappa \) the von Karman constant (\( \kappa = 0.4 \)), \( u_* \) the friction velocity corresponding to the total bed shear stress, and \( a \) the reference elevation above the bed elevation.

6.2 Two-dimensional sediment transport equation

The two-dimensional (2D) sediment transport equation for the depth-averaged suspended-load concentration \( C = C(x,y,t) \) is obtained by integrating the 3D sediment transport equation over the suspended-load zone. By applying the Leibniz integral rule, adopting suitable boundary conditions and assuming that the bedload zone is very thin, the depth-integrated suspended-load transport equation is obtained:

\[ \frac{\partial hC}{\partial t} + \frac{\partial (hUC)}{\partial x} + \frac{\partial (hVC)}{\partial y} = \frac{\partial}{\partial x} \left( h \epsilon_s \frac{\partial C}{\partial x} \right) + \frac{\partial}{\partial y} \left( h \epsilon_s \frac{\partial C}{\partial y} \right) + E - D, \]

with \( h = Z_s - Z_f \approx Z_s - Z_{ref} \) the water depth, assuming that the bedload layer thickness is very thin, \((U, V)\) are the depth-averaged velocity components in the \( x \)- and \( y \)-directions, respectively. The net sediment flux \( E - D \) is determined based on the concept of equilibrium concentration, see [11]:

\[ (E - D)_{z_{ref}} = w_s \left( C_{eq} - C_{ref} \right) \]

where \( C_{eq} \) is the equilibrium near-bed concentration and \( C_{ref} \) is the near-bed concentration, calculated at the interface between the bed-load and the suspended load, \( z = Z_{ref} \). The reference elevation \( Z_{ref} \) corresponds to the interface between bedload and suspended load.

Equation (34), can be expressed in its non-conservative form as:

\[ \frac{\partial C}{\partial t} + U \frac{\partial C}{\partial x} + V \frac{\partial C}{\partial y} = \frac{1}{h} \left( \frac{\partial}{\partial x} \left( h \epsilon_s \frac{\partial C}{\partial x} \right) + \frac{\partial}{\partial y} \left( h \epsilon_s \frac{\partial C}{\partial y} \right) \right) + \frac{E - D}{h}, \]

where \( U \) and \( V \) are the depth-averaged convective flow velocities in the \( x \)- and \( y \)-directions, respectively.

Keywords
- SUSPENSION = NON, default value
In Equation (34), the net sediment flux $E - D$ is expressed in $\text{m s}^{-1}$. For consistency with bedload units, expressed in $\text{m}^2 \text{s}^{-1}$, concentration is dimensionless. However, the user can choose concentration by mass per unit volume of the mixture ($\text{kg m}^{-3}$) by using the keyword MASS CONCENTRATION. The relation between concentration by volume and concentration by mass is $C_m (\text{kg m}^{-3}) = \rho_s C$.

### 6.3 Convection velocity

In Equation (34) it is supposed that $Q_{sx} = h U C$ and $Q_{sy} = h V C$. A straightforward treatment of the advection terms would imply the definition of an advection velocity and replacement of the depth-averaged velocities in Eq. (34). For the $x-$ component:

$$Q_{sx} = h U_{\text{conv}} C$$

A correction factor is introduced in Sisyphè, defined by:

$$F_{\text{conv}} = \frac{U_{\text{conv}}}{U}$$

The convection velocity should be smaller than the mean flow velocity ($F_{\text{conv}} \leq 1$) since sediment concentrations are mostly transported in the lower part of the water column where velocities are smaller. We further assume an exponential profile concentration profile which is a reasonable approximation of the Rouse profile, and a logarithmic velocity profile, in order to establish the following analytical expression for $F_{\text{conv}}$:

$$F_{\text{conv}} = -\frac{l_2 - \ln \left( \frac{B_3}{B_1} \right) l_1}{l_1 \ln \left( \frac{B_3}{B_1} \right)}$$

with $B = k_s / h = Z_{\text{ref}} / h$ and

$$l_1 = \int_B^1 \left( \frac{1-u}{u} \right)^R du, \quad l_2 = \int_B^1 \ln u \left( \frac{1-u}{u} \right)^R du.$$  

A similar treatment is done for the $y-$component. For further details, see [27].

### Keywords

- CORRECTION ON CONVECTION VELOCITY (= NO, default option)

### 6.4 Numerical treatments

#### 6.4.1 Treatment of the diffusion terms

According to the choice of the parameter OPDTRA of the keyword OPTION FOR THE DIFFUSION OF TRACER, the diffusion terms in (36) can be simplified as follows:

- OPDTRA = 1 : the diffusion term is solved in the form $\nabla \cdot (\epsilon_s \nabla T)$
- OPDTRA = 2 : the diffusion term is solved in the form $\frac{1}{h} \nabla \cdot (h \epsilon_s \nabla T)$

The value of the dispersion coefficient $\epsilon_s$ depends on the choice of the parameter OPTDIF of the keyword OPTION FOR THE DISPERSION:

- OPTDIF = 1 : the values of the constant longitudinal and transversal dispersivity coefficients ($T1$ and $T2$ respectively) are provided with the keywords DISPERSION ALONG THE FLOW and DISPERSION ACROSS THE FLOW
- OPTDIF = 2: the values of the longitudinal and transversal dispersion coefficients are computed with the Elder model $T_1 = \alpha_l u^* h$ and $T_2 = \alpha_t u^* h$, where the coefficients $\alpha_l$ and $\alpha_t$ can be provided with the keywords DISPERSION ALONG THE FLOW and DISPERSION ACROSS THE FLOW.
- OPTDIF = 3: the values of the dispersion coefficients are provide by TELEMAC-2D

The diffusion term can also be set to zero with keyword DIFFUSION = NO (YES is the option by default).

**Attention**

Different schemes are available for solving the non-linear advection terms depending on the choice of the parameter the classical characteristics schemes. To the diffusive schemes SUPG and PSI, it is recommended to use conservative schemes.

### 6.4.2 Treatment of the advection terms

The choice of the numerical scheme is based on keyword TYPE OF ADVECTION. The following numerical methods are available in Sisyphe:

- **Method of characteristics**
  - Unconditionally stable and monotonous
  - Diffusive for small time steps
  - Not mass conservative

- **Method Streamline Upwind Petrov Galerkin SUPG**
  - Courant number criteria
  - Not mass conservative
  - Less diffusive for small time steps

- **Conservative N-scheme (similar to finite volumes)**
  - Solves the continuity equation under its conservative form
  - Recommended when the correction on convection velocity is accounted for
  - Courant number limitation (sub-iterations are included to reduce the time step
  - Numerical schemes 13 and 14 are the same as 3 and 4 but here adapted to the presence of tidal flats based on positive water depth algorithm

- **Distributive schemes (PSI)**
  - like scheme 4, but the fluxes are corrected according to the tracer value: this relaxes the courant number criteria and it is also less diffusive than scheme 4 and 14
  - The CPU time is however increased
  - This method should not applied for tidal flats

**Attention**

It is recommended to use scheme 4 or 14 (if tidal flats are present) as a good compromise (accuracy/computational time)

**Keywords**

- TETA SUSPENSION (= 1, default option)
- TYPE OF ADVECTION (= 1, default option)
- SOLVER FOR SUSPENSION (= 3, conjugate gradient by default option)
- PRECONDITIONING FOR SUSPENSION (= NO, default option)
- SOLVER ACCURACY FOR SUSPENSION ($1.0 \times 10^{-8}$, default option)
- MAXIMUM NUMBER OF ITERATIONS FOR SOLVER FOR SUSPENSION (= 50, default option)
- OPTION FOR THE DISPERSION (= 1, constant dispersion coefficient by default option)
- DISPERSION ALONG THE FLOW ($1.0 \times 10^{-2}$, default option)
- DISPERSION ACROSS THE FLOW ($1.0 \times 10^{-2}$, default option)
6.5 Equilibrium concentrations

6.5.1 Erosion and deposition rates

For non-cohesive sediments, the net sediment flux \( E - D \) is determined based on the concept of equilibrium concentration, see [11]:

\[
(E - D)_{Z_{ref}} = w_s \left( C_{eq} - C_{ref} \right)
\]  

(37)

where \( C_{eq} \) is the equilibrium near-bed concentration and \( C_{ref} \) is the near-bed concentration, calculated at the interface between the bed-load and the suspended load, \( z = Z_{ref} \). The reference elevation \( Z_{ref} \) corresponds to the interface between bedload and suspended load. For flat beds, the bed-load layer thickness is proportional to the grain size, whereas when the bed is rippled, the bedload layer thickness scales with the equilibrium bed roughness \( (k_r) \). The definition of the reference elevation needs also to be consistent with the choice of the equilibrium near-bed concentration formula.

The keyword \texttt{REFERENCE CONCENTRATION FORMULA} access to four different semi-empirical formulas:

- ICQ= 1 : Zyserman and Fredsoe formula

\[
C_{eq} = \frac{0.331(\theta' - \theta_c)^{1.75}}{1 + 0.72(\theta' - \theta_c)^{1.75}}
\]

(38)

where \( \theta_c \) is the critical Shields parameter and \( \theta' = \mu \theta \), the non-dimensional skin friction which is related to the Shields parameter, by use of the ripple correction factor. According to Zyserman and Fredsoe, the reference elevation should be set to \( Z_{ref} = 2d_{50} \). In Sisyphe we take \( Z_{ref} = k'_s \), where \( k'_s \) is the skin bed roughness for flat bed \( (k'_s \approx d_{50}) \), the default value of proportionality factor is \texttt{KSPRATIO =3}.

- ICQ= 2 : Bijker (1992) formula

The equilibrium concentration corresponds to the volume concentration at the top of the bed-load layer. It can related to the bed load transport rate:

\[
C_{eq} = \frac{Q_s}{bZ_{ref}u_*}
\]

(39)

with an empirical factor \( b = 6.34 \).

According to Bijker, \( Z_{ref} = k_r \).

\[\text{Attention}\]

The near bed concentration computed with the Bijker formula is related to the bedload. Therefore, this option cannot be used without bedload transport (\texttt{BED-LOAD = YES}). Furthermore, both bedload and suspended load transport must be calculated at each time step, with the keyword \texttt{PERIOD OF COUPLING} set equal to 1.

- ICQ= 3 : van Rijn formula

\[
C_{eq} = 0.015d_{50}(\tau_p/\tau_c - 1)^{3/2} Z_{ref}D_{0.3}
\]

(40)

The concentration is defined at \( Z_{ref} = \max(k_s/2; 0.01m) \)

- ICQ= 4 : Soulsby-van Rijn formula

For details, we refer to the reference [42].

6.5.2 Ratio between the reference and depth-averaged concentration

By depth-integration of the Rouse profile (41), the following relation can be established between the mean (depth-averaged) concentration and the reference concentration:

\[ C_{ref} = FC \]
where:

$$F^{-1} = \left( \frac{Z_{\text{ref}}}{h} \right)^R \int_{Z_{\text{ref}}/h}^{1} \left( \frac{1-u}{u} \right)^R du. \quad (41)$$

In Sisyphes, we use the following approximate expression for $F$:

$$F^{-1} = \frac{1}{1-Z} B^R \left( 1 - B^{(1-R)} \right), \quad \text{for} \ R \neq 1$$

$$F^{-1} = -B \log B, \quad \text{for} \ R = 1,$$

with $B = Z_{\text{ref}}/h$.

### Keywords

- REFERENCE CONCENTRATION FORMULA = 1, default value

### 6.6 Initial and boundary conditions for sediment concentrations

#### 6.6.1 Initial conditions

The initial values of volume concentration for each class can be either imposed in the subroutine `condim_susp.f` or specified in the steering file through the keyword INITIAL SUSPENSION CONCENTRATIONS.

**Attention**

It will not be used if EQUILIBRIUM INFLOW CONCENTRATION = YES.

#### 6.6.2 Boundary conditions

For boundary conditions, the concentration of each class can be specified in the steering file through keyword CONCENTRATION PER CLASS AT BOUNDARIES. To avoid unwanted erosion or sedimentation at the entrance of the domain, it may be also convenient to use keyword EQUILIBRIUM INFLOW CONCENTRATION = YES to specify the value of the concentration at inflow, according to the choice of the REFERENCE CONCENTRATION FORMULA. The depth-averaged equilibrium concentration is then calculated assuming equilibrium concentration at the bed and a Rouse profile correction for the $F$ factor. Input concentrations can be also directly specified in the subroutine `conlit.f`. In `conlit.f`, it is assumed that LIEBOR = LICBOR.
7 Bed evolution equation

7.1 Bedload

To calculate the bed evolution, Sisyphe solves the Exner equation:

\[
(1 - n) \frac{\partial Z_f}{\partial t} + \nabla \cdot Q_b = 0, \tag{42}
\]

where \( n \) is the non-cohesive bed porosity (specified by the keyword \texttt{NON COHESIVE BED POROSITY}, = 0.40 by default), \( Z_f \) the bottom elevation, and \( Q_b \) (m\(^2\)s\(^{-1}\)) the solid volume transport (bedload) per unit width.

Equation (42) states that the variation of sediment bed thickness can be derived from a simple mass balance and it is valid for equilibrium conditions.

7.2 Boundary conditions

From v6.2, as a general case the user has to specify two different boundary conditions files: a conlim file for the hydrodynamics module (e.g., for Telemac-2d) and a different conlim file for Sisyphe. This allows the user to apply different conditions for a tracer in Telemac-2d and for bed evolution or concentration in Sisyphe. However, in most simple applications, the same boundary condition can be used.

In the sediment transport boundary condition file, the following boundary condition types can be imposed on the open boundaries:

**Imposed bed evolution**

For this case, LIEBOR=5 (KENT) and LIQBOR=4 (KSORT) for boundaries with prescribed variation of elevation. For example, if the user wants to define a boundary with a fixed bed elevation (zero evolution), the conlim file should look like:

```
(not used) LIQBOR (not used) ... LIEBOR EBOR
(4) 4 (5) ... 5 0.
```

**Imposed sediment transport rates**

For this case, LIEBOR = 4 (KSORT) and LIQBOR = 5 (KENT). For example, if the user wants to define a boundary with an imposed transport rate \( = 1.0 \times 10^{-5} \) m\(^2\)s\(^{-1}\), the conlim file should look like:

```
(not used) LIQBOR (not used) Q2BOR ... LIEBOR EBOR
(+) 5 (+) 1.E-5 ... 4 0.
```

For this case, a boundary condition file for Sisyphe is needed. LIHBOR is actually not used but is now available for users. LIQBOR is the boundary condition on solid discharge. Q2BOR is the prescribed solid discharge given at boundary points, and expressed in m\(^2\)s\(^{-1}\).

Alternatively, the user can give the width-integrated values of the imposed sediment transport along the boundaries by using the keyword \texttt{PRESCRIBED SOLID DISCHARGES}, which is in m\(^3\)s\(^{-1}\). When the keyword \texttt{PRESCRIBED SOLID DISCHARGES} is given it supersedes Q2BOR, which is then taken as a constant profile.

For non-uniform sediment distribution, the imposed values of transport rate are distributed to every class with the help of the fractions (array AVAIL), this is done for EBOR and QBOR in conlis.f and may be changed.
7.3 Treatment of rigid beds and tidal flats

Non-erodable beds are treated numerically by limiting bedload erosion and letting incoming sediment pass over. The problem of rigid beds is conceptually trivial but numerically complex.

For finite elements the minimum water depth algorithm allows a natural treatment of rigid beds, see [25]. The sediment is managed as a layer with a depth that must remain positive, and the Exner equation is solved similarly to the shallow water continuity equation in the subroutine positive_depths.f.

For finite volume schemes, the treatment must be explicitly specified and different options are available with the keyword OPTION FOR THE TREATMENT OF NON ERODABLE BEDS:

- OPTBAN = 0: erodable bottoms everywhere (default option)
- OPTBAN = 1: minimisation of the solid discharge
- OPTBAN = 2: nul solid discharge
- OPTBAN = 3: minimisation of the solid discharge in FINITE ELEMENTS / MASS-LUMPING
- OPTBAN = 4: minimisation of the solid discharge in FINITE VOLUMES

Attention

For the finite element method, the treatment of rigid bed is implicitly done by the numerical scheme and previous options are now obsolete.

The space location and position of the rigid bed can be changed in the subroutine noerod.f. By default, the position of the rigid bed is located at \( z = -100 \text{m} \).

7.4 Tidal flats

Tidal flats are areas of the computational domain where the water depth can become zero during the simulation.

For finite elements the minimum water depth algorithm allows a natural treatment of tidal flats, see [25] and the Exner equation is solved similarly to the shallow water continuity equation in the subroutine positive_depths.f.

For finite volume schemes, tidal flats can be treated with the keyword TIDAL FLATS. The companion keyword OPTION FOR THE TREATMENT OF TIDAL FLATS allows to choose between two different approaches:

- OPTBAN = 1: is the default option, and the equations are solved everywhere. The water depth is set to a minimum value, controlled by the keyword MINIMAL VALUE OF THE WATER HEIGHT, with a default value = \( 1.0 \times 10^{-3} \text{m} \).
- OPTBAN = 2: this option removes from the computational domain the elements with points that present a water depth less than a minimum value. This option should be avoided because it is not exactly mass-conservative.
### Keywords
- TIDAL FLATS (= YES, default option)
- MINIMAL VALUE OF THE WATER HEIGHT (= 1.D-3m, default value)
- OPTION FOR THE TREATMENT OF TIDAL FLATS (OPTBAN = 1, default option)

### Attention
For the finite element method, the treatment of tidal flats is implicitly done by the numerical scheme and previous options are now obsolete.

#### 7.5 Bed evolution for suspension

By considering only suspended load sediment transport, the bed evolution is function of the net vertical sediment flux at the interface between the bedload and the suspended load given by:

\[
(1 - n) \frac{\partial Z_f}{\partial t} + (E - D)_{z=Z_{ref}} = 0
\]

with \(Z_f\) the bed elevation, \(Z_{ref}\) the interface between the bedload and suspended load, and \(n\) the bed porosity. Once the flow variables are determined by solving the hydrodynamics and suspended sediment transport equations, changes of bed level are computed from Equation (43) for the cell coincident to the bed, calculating at each time step the net sediment flux \(E - D\) at the bedload-suspended load interface \((z = Z_{ref})\), as explained later. Further details on the derivation of the mass balance equation (43) and coupling strategies can be found in [65].
8 Wave effects

8.1 Introduction

In coastal zones, the effect of waves superimposed to a mean current (wave-induced or tidal), modifies the bed structure. Due to the reduced thickness of the bed boundary layer, the bottom shear stress increases largely and the resulting sand transport rate could be in many cases of one order of magnitude greater than in the case of currents alone. Furthermore, the wave-generated ripples may also have a strong effect on the bed roughness and sand transport mechanisms. In Sisyphé, these effects can be incorporated into the numerical simulations when the keyword EFFECT OF WAVES is activated.

To compute sediment transport rates due to the action of waves, the wave height, period and direction need to be specified. This information can be provided from a fortran file, by the subroutine `condim_sisyphé.f`, from a file containing the variables computed previously by the wave module, e.g. `tomawac`, or by internal coupling.

The wave orbital velocity $U_0$ is calculated by Sisyphé by assuming the validity of the linear theory:

$$U_0 = \frac{H_s \omega}{2 \sinh(kh)},$$

where $H_s$ is the significant wave height, $\omega = 2\pi/T_p$ is the intrinsic angular frequency, $k = 2\pi/L$ is the wave number, with $L$ the wave length. The wave number is calculated from the dispersion relation (neglecting surface tension and amplitude effects) as:

$$\omega^2 = gk \tanh(kh).$$

8.1.1 Wave information

The wave information, required as input parameters for the morphodynamic model, are the significant wave height $H_s$, the peak wave period $T_p$ and the wave direction $\theta_w$. If $\theta_w$ is not provided, Sisyphé assumes the default value $\theta_w = 90^\circ$, which means that the direction of propagation of the wave is parallel to the $x$-axis. When using the Bailard’s transport formula, this variable must be always provided.

If the wave field is given from a file containing the variables computed previously on the same mesh by the wave module, e.g. `tomawac`, the keyword WAVE FILE must be specified in the Sisyphé steering file. This file must contain the significant wave height $H_s$ ($HM0$), the wave peak period $T_p$ ($TPR5$) and mean wave direction $\theta_w$ relative to $y$-axis ($DMOY$).

⚠️ **Attention**

For the keyword WAVE FILE, variables are given by the last record of the file.

If the currents and wave field is provided from a file containing the variables computed previously on the same mesh by the currents and wave modules, e.g. `telemac-2d` and `tomawac`, respectively, the keyword HYDRODYNAMIC FILE must be specified in the Sisyphé steering file. This file must contain the hydrodynamic variables (water height and velocity field) as well as the wave variables: significant wave height $H_s$ ($HM0$), wave peak period $T_p$ ($TPR5$) and mean wave direction $\theta_w$ relative to $y$-axis ($DMOY$). The users has also the choice of provide the wave variables from the subroutine `condim_sisyphé.f`.

⚠️ **Attention**

For the keyword HYDRODYNAMIC FILE, the variables are given by the last record of the file if the case is steady or by the time steps describing the tide or flood for the unsteady case.
8.2 Wave-induced bottom friction

8.2.1 Wave-induced bottom friction

The maximum stress due to waves is calculated at each time step as a function of the wave-orbital velocity \( U_w \) by using the quadratic friction coefficient \( f_w \) due to waves:

\[
\tau_w = \frac{1}{2} \rho f_w U_w^2.
\]

The wave friction factor \( f_w \) is calculated as a function of the orbital amplitude on the bed \( A_0 = U_w/\omega \) and the bed roughness \( k_s \):

\[
f_w = f_w\left(\frac{A_0}{k_s}\right),
\]

In Sisyphë, the formula proposed by Swart \textit{et al.} [43] is provided:

\[
f_w = \begin{cases} 
\exp\left(-6.0 + 5.2 \left(\frac{A_0}{k_s}\right)^{-0.19}\right), & \text{if } A_0/k_s > 1.57 \\
0.30, & \text{otherwise}
\end{cases}
\]

8.2.2 Wave-current interaction

For combined waves and currents, the wave-induced (mean and maximum) bottom stresses can be of one order of magnitude larger than in the case of currents alone. Different models can be found in the literature to calculate the wave and current bottom stresses \( \tau_{cw} \), as a function of the bottom shear stress due to currents only \( \tau_c \) and the maximum shear stress due to waves only \( \tau_w \). Following Bijker [7]:

\[
\tau_{cw} = \tau_c + \frac{1}{2} \tau_w. \tag{44}
\]

The mean \( \tau_{\text{mean}} \) and maximum \( \tau_{\text{max}} \) shear stresses can be computed following the Soulsby’s method [42]. Non-dimensional variables are defined:

\[
X = \frac{\tau_0}{\tau_0 + \tau_w}; \quad Y = \frac{\tau_{\text{mean}}}{\tau_0 + \tau_w}; \quad Z = \frac{\tau_{\text{max}}}{\tau_0 + \tau_w}
\]

They can be parameterized as:

\[
Y = X \left(1 + bX^p(1 - X)^q\right),
\]

\[
Z = 1 + aX^m(1 - X)^n
\]

The various model coefficients \((a, b, m, n, p, q)\) are empirical functions of friction coefficients \( f_w \) and \( C_D \), and the wave-current angle \( \phi \):

\[
a = (a_1 + a_2 |\cos \phi|^l) + (a_3 + a_4 |\cos \phi|^l) \log_{10}\left(\frac{2f_w}{C_D}\right),
\]

\[
b = (b_1 + b_2 |\cos \phi|^l) + (b_3 + b_4 |\cos \phi|^l) \log_{10}\left(\frac{2f_w}{C_D}\right),
\]

......

with similar expressions for \(m, n, p, \) and \( q \). The fitted constants \((a_i, b_i, m_i, n_i, p_i, q_i, I \text{ and } f)\) depend on the turbulence model selected. Table 2 shows the coefficients used in Sisyphë corresponding to the model of Huynh-Thanh and Temperville [28] for \( I = 0.82, f = 2.7 \).
8.2.3 Friction coefficient under combined waves and currents

The characteristic shear stress $\langle |\tau_{cw}(t)| \rangle$, is related to the time-averaged mean velocity:

$$\langle |\tau_{cw}(t)| \rangle = \rho f_{cw} \left\langle |\overrightarrow{U(t)}|^2 \right\rangle,$$

where

$$\left\langle |\overrightarrow{U(t)}|^2 \right\rangle = U_c^2 + \frac{1}{2} U_w^2.$$

According to Camenen [8], the characteristic shear stress is taken as a weighted average between $\tau_{\text{mean}}$ and $\tau_{\text{max}}$:

$$\langle |\tau_{cw}(t)| \rangle = X \tau_{\text{mean}} + (1 - X) \tau_{\text{max}},$$

which is equivalent to:

$$\langle |\tau_{cw}(t)| \rangle = Y \tau_c + Z \tau_w.$$

Finally, the expression for the wave-current interaction factor is:

$$f_{cw} = \frac{Y \tau_c + Z \tau_w}{U_c^2 + \frac{1}{2} U_w^2}. \quad (45)$$

This expression is used by Bailard [2] and Dibajnia and Watanabe [14] to compute sand transport rates.

8.3 Wave-induced ripples

Equilibrium ripples are generally observed outside the surf zone. Their dimensions can be predicted as a function of waves (orbital velocity $U_0$ and wave period $T = \frac{2\pi}{\omega}$), for a given uniform sediment diameter $d_{50}$. In Sisyphé, the procedure of Wiberg and Harris has been implemented [60]. This formulation is only applicable to oscillatory flow conditions and does not account for the effect of a superimposed mean current.

Ripples can be classified into three types depending on the value of the ratio of wave orbital diameter to mean grain diameter $D_0/d_{50}$, with $D_0 = 2U_0/\omega$. The ripples wave length $\lambda$ and height $\eta$, can be estimated as follows:

- Under moderate wave conditions (orbital regime), the ripple dimensions are proportional to $D_0$:
  $$\lambda = 0.62 D_0, \quad \eta = 0.17 \lambda.$$

- For larger waves (anorbital regime), ripple dimensions scale with the sand grain diameter:
  $$\lambda = 535 d_{50}, \quad \eta = \lambda \exp \left( -0.095 \left( \log \frac{D_0}{\eta} \right)^2 + 0.042 \log \frac{D_0}{\eta} - 2.28 \right).$$
The effect of ripples is to increase the bed roughness. Based on the Bijker’s formula, in Sisyphe it is assumed that:

$$k_s = \max(\eta, 3d_{50})$$.

Following Tanaka and Dang [49], the effect of the mean current superimposed to the waves is accounted by introducing a correction factor $$\alpha$$ to the orbital velocity:

$$\alpha = 1 + 0.81 \left( \tanh(0.3S^2/3) \right)^{2.5} \left( \frac{U}{U_w} \right)^{1.9}$$,

with $$S_* = d_{50} \sqrt{(s-1)gd_{50}/4\nu}$$.

### 8.4 Wave-induced sediment transport

The effect of waves modifies the sand transport rates when it is superimposed to currents. This effect can be accounted by using one of the following wave sand transport formula implemented in Sisyphe, namely the Bijker’s formula [7], the Soulsby-van Rijn’s formula [42], the Bailard’s formula [2] and the Dibajnia and Watanabe’s formula [14].

The keyword **BED-LOAD TRANSPORT FORMULA** allows the user to choose:

- ICF = 4: the Bijker’s formula (1968) can be used to determine the total transport rate $$Q_s$$ (bed-load + suspension), with its two components the bedload $$Q_{sc}$$ and the suspended load $$Q_{ss}$$ computed separately. For bedload transport, Bijker extended the steady bed-load formula proposed by Frijlink (1952), by adding the wave effects. In non-dimensional form, the bedload transport rate is:

  $$\Phi_b = b\theta_c^{0.5} \exp\left(-0.27 \frac{1}{\mu \theta_{cw}}\right),$$

  where $$\theta_c$$ is the non-dimensional shear stress due to currents alone, $$\theta_{cw}$$ is the non-dimensional shear stress due to wave-current interaction, and $$\mu$$ is a correction factor which accounts for the effect of ripples. The shear stress under combined wave and current is calculated by Equation (44).

In the original formulation, the coefficient $$b = 5$$. Recent studies showed that $$b = 2$$ provides better results when comparing with field and experimental data. By default, in Sisyphe $$b = 2$$ but this value can be modified with the keyword **B VALUE FOR THE BIJKER FORMULA**. The ripple factor correction $$\mu$$ is calculated in the same way as in the Meyer-Peter-Mueller formula for currents only.

The suspended load transport is solved in a simplified manner, by assuming the concentration profile to be in equilibrium. The inertia effects are not modeled and it is assumed that no exchange takes place with the bed load layer.

After depth-integration and by assuming a Rouse profile for the concentration and a logarithmic velocity profile for the mean velocity profile, the suspended load can be written as:

$$Q_{ss} = Q_{sc} I,$$

where

$$I = 1.83 \times 0.216 \frac{B^{A-1}}{(1-B)^A} \int_B^1 \left( \frac{1-y}{y} \right)^A \ln\left( \frac{33y}{B} \right) dy,$$

with

$$A = \frac{W_s}{k_{II_s}}, \quad u_* = \sqrt{\frac{\tau_{cw}}{\rho}}, \quad B = k_s/h.$$
- **ICF = 5**: Soulsby-van Rijn formula, the transport rate due to the combined action of waves and current is provided by the following equation:

\[
Q_{b,s} = A_{b,s} U \left[ \left( U^2 + 2\frac{0.018}{C_D} U_0^2 \right)^{0.5} - U_{cr} \right]^{2.4}.
\]

This formula can be applied to estimate the components of the total sand transport rate (bedload \(Q_b\) and suspended load \(Q_s\)), and it is suitable for beds covered by ripples. The bedload and suspended load coefficients \(A_{b,s}\) are computed as:

\[
A_b = \frac{0.005 h \left( d_{50}/h \right)^{1.2}}{((s-1)g d_{50})^{1.2}},
\]

\[
A_s = \frac{0.012 d_{50} D_{90}^{-0.6}}{((s-1)g d_{50})^{1.2}},
\]

where \(U\) is the depth-averaged current velocity, \(U_0\) is the RMS orbital velocity of waves, and \(C_D\) is the quadratic drag coefficient due to current alone. This formula has been validated assuming a rippled bed roughness with \(k_s = 0.18\) m. The critical entrainment velocity \(U_{cr}\) is given by the expression:

\[
U_{cr} = \left\{ \begin{array}{ll}
0.19 d_{50}^{0.3} \log_{10} \left( \frac{4h}{D_{90}} \right), & \text{if } 0.1\text{mm} \leq d_{50} \leq 0.5\text{mm} \\
8.5 d_{50}^{0.6} \log_{10} \left( \frac{4h}{D_{90}} \right), & \text{if } 0.5\text{mm} \leq d_{50} \leq 2.0\text{mm}.
\end{array} \right.
\]

The diameter \(D_{90}\), characteristic of the coarser grains, is specified with the keyword \(D90\). The validity range for the Soulsby-van Rijn formula is \(h = [1-20] \text{ m}, U = [0.5-5] \text{ ms}^{-1}\), and \(d_{50} = [0.1-2.0] \text{ mm}\).

- **ICF = 8**: the Bailard’s formula is based on the energetic approach. The bedload and the suspended load components of the sand transport rate are expressed respectively as the third- and fourth-order momentum of the near-bed time-varying velocity field \(\vec{U}(t)\), as follows:

\[
Q_b = f_{cw} \frac{\epsilon_c}{g (s-1) \tan \varphi} \langle |\vec{U}|^2 \vec{U} \rangle,
\]

\[
Q_s = f_{cw} \frac{\epsilon_s}{g (s-1) W_s} \langle |\vec{U}|^3 \vec{U} \rangle,
\]

with \(f_{cw}\) the friction coefficient which accounts for wave-current interactions, \(\epsilon_s = 0.02, \epsilon_c = 0.1\) empirical factors, \(\varphi\) sediment friction angle (\(\tan \varphi = 0.63\)) and \(< \cdot \rangle\) time-averaged over a wave-period.

Under combined wave and currents, the time-varying velocity vector \(\vec{U}(t)\) is composed of a mean component \(U_c\) and an oscillatory component of amplitude \(U_0\), assuming linear waves; \(\phi\) is the angle between the current and the wave direction, \(\phi_c\) is the angle between the mean current direction and the \(x\)-axis, and \(\phi_w\) is the angle between the wave direction of propagation and the \(x\)-axis. The time-varying velocity field verifies:

\[
\vec{U}(t) = U_c \exp^{i \phi_c} + U_0 \cos \omega t \exp^{i \phi_w}.
\]

For the third-order term, one can derive an analytical expression in the general case, whereas for the fourth-order momentum, we have to assume the waves and currents to be co-linear (\(\phi = 0\)), in order to find an analytical expression. For the general case of a non-linear non-colinear wave, we would have to integrate numerically the fourth-order velocity term [8]. However, this method has showed to be inefficient. Expressions for the third and fourth-order terms are given below:
- Third-order term:

\[
\begin{align*}
\left\langle \left| \vec{u} \right| \rightangle^2 = & \overline{u}_c (U_c^2 + \frac{1}{2} U_0^2) + \overline{u}_0 (\overline{u}_c \cdot \overline{u}_0) \\
\left\langle \left| \vec{u} \right|^2 \rightangle_x = & \left[ U_c^3 + U_c U_0^3 (1 + \frac{1}{2} \cos 2\phi) \right] \cos \phi_c - \frac{1}{2} U_c U_0^3 \sin 2\phi \sin \phi_c \\
\left\langle \left| \vec{u} \right|^2 \rightangle_y = & \left[ U_c^3 + U_c U_0^3 (1 + \frac{1}{2} \cos 2\phi) \right] \sin \phi_c + \frac{1}{2} U_c U_0^3 \sin 2\phi \cos \phi_c 
\end{align*}
\]

- Fourth-order term:

\[
\begin{align*}
\left\langle \left| \vec{u} \right|^3 \rightangle_x = & \frac{1}{8} (24 U_0^2 U_c^2 + 8 U_c^4 + 3 U_0^4) \cos \phi_c \\
\left\langle \left| \vec{u} \right|^3 \rightangle_y = & \frac{1}{8} (24 U_0^2 U_c^2 + 8 U_c^4 + 3 U_0^4) \sin \phi_c 
\end{align*}
\]

- ICF = 9: the Dibajnia and Watanabe (1992) formula is an unsteady formula, which accounts for inertia effects. The sand transport rate is calculated by the expression:

\[
\vec{Q}_s = \alpha \left| \Gamma \right|^1 \beta
\]

with \( \alpha = 0.0001 \) and \( \beta = 0.55 \) empirical model coefficients and \( \left| \Gamma \right|^1 \beta \) the difference between the amounts of sediments transported in the onshore and offshore directions. This formula is used to estimate the intensity of the solid transport rate, the direction is assumed to follow the mean current direction. In the wave direction, the time-varying velocity is given by:

\[
U(t) = U_c \cos \phi + U_0 \sin \omega t,
\]

\( r \) is defined by the asymmetry parameter:

\[
r = \frac{U_0}{U_c \cos \phi}.
\]

The wave cycle is decomposed into two parts:

1. During the first part of the wave-cycle (0 < t < T_1), the velocities are directed onshore (U(t) > 0).
2. During the second part (T_2 = T - T_1), the velocities are negatives (U(t) < 0).

The sand transport rate in the wave direction is:

\[
\Gamma_x = \frac{u_1 T_1 (\Omega_1^3 + \Omega_2^3) - u_2 T_2 (\Omega_3^3 + \Omega_4^3)}{(\Omega_1 + \Omega_2) T}
\]

where \( \Omega_1 \) and \( \Omega_2 \) represent the amount of sand transported in the onshore and offshore directions which will be deposited before flow reversal, respectively, \( \Omega_3 \) and \( \Omega_4 \) represent the remaining part which stay in suspension after flow reversal. The inertia of sediment depends on the ratio \( d_{50}/w_s \), and represented by parameter \( \omega_i \):

\[
\omega_i = \frac{u_i^2}{2 (s - 1) g W_s T_i}
\]

with

\[
u_i^2 = \frac{2}{T_i} \int_{u > 0 < u_0} u^2 dt,
\]

and two different cases:
1. All sediment is deposited before flow reversal $\omega_i \leq \omega_{\text{crit}}$

$$\Omega_i = \omega_i \frac{2W_s T_i}{d_{50}}, \quad \Omega'_i = 0$$

2. Part of the sediment stays in suspension after flow reversal $\omega_i \geq \omega_{\text{crit}}$

$$\Omega_i = \omega_{\text{crit}} \frac{2W_s T_i}{d_{50}}, \quad \Omega'_i = \left(\omega_i - \omega_{\text{crit}}\right) \frac{2W_s T_i}{d_{50}}.$$  

The critical value of $\omega_{\text{crit}}$ is calculated as function of the wave-current non-dimensional friction:

$$\Theta_{\text{cw}} = \frac{\langle \tau_{\text{cw}} \rangle}{\rho (s-1)g d_{50}}.$$  

According to Temperville et Guiza:

$$\omega_{\text{crit}} = \begin{cases} 
0.03, & \text{if } \Theta_{\text{cw}} \leq 0.2 \\
1 - \sqrt{1 - \left(\Theta_{\text{cw}} - 0.2\right)/0.58}^2, & \text{if } 0.2 \leq \Theta_{\text{cw}} \leq 0.4 \\
0.8104 \sqrt{\Theta_{\text{cw}} - 0.4225}, & \text{if } 0.4 \leq \Theta_{\text{cw}} \leq 1.5 \\
0.7236 \sqrt{\Theta_{\text{cw}} - 0.3162}, & \text{otherwise}
\end{cases}$$

**Attention**

In **Sisyphe**, the solid discharge is assumed to be oriented in the direction of the mean current. The possible deviation of the transport in the direction of waves, which can be important in the near shore zone due to non-linear effects, is not accounted for.

### 8.5 Internal coupling waves-currents and sediment transport

The internal coupling waves-currents and sediment transport is implemented in the Telemac-Mascaret system. The keyword **COUPLING WITH = 'TOMAWAC, SISYPHE'** (in the Telemac-2d steering file) is used to internally couple with **TOMAWAC** and **SISYPHE**.

The coupling period with **TOMAWAC** and **SISYPHE** can be controlled with the keywords **COUPLING PERIOD FOR TOMAWAC** and **COUPLING PERIOD FOR SISYPHE**, respectively.

In the **TELEMAC-2D** cas file, the keyword **WAVE DRIVEN CURRENTS = YES** allows to incorporate the effect of waves in the hydrodynamic simulation.

In the Sisyphe cas file, the keyword **EFFECT OF WAVES = YES** is used to consider the effect of the waves on the solid transport formula. In **Sisyphe**, the implemented bedload transport formulae that consider the combined effect of currents and waves are Bijker (=4), Soulsby-van Rijn (=5), Bailard (=8) and Dibajnia and Watanabe (=9). Formulae 4, 5 and 8 consider bedload and suspended load transport. Formula 9 considers total transport.
9 Sand grading effects

9.1 Sediment bed composition

9.1.1 Granulometry distribution

The number of classes of bed material is specified in the steering file with the keyword NUMBER OF SIZE-CLASSES OF BED MATERIAL and is limited to \text{NSICALA} \leq 20. For uniform sediments, the granulometry distribution is represented by one single value \text{NSICALA} = 1 for the whole domain. The mean grain diameter and corresponding settling velocity are defined in the steering file.

For sediment mixtures, the granulometry distribution is discretized in a number of classes. Each class of sediment \( 1 < j \leq \text{NSICALA} \) is defined by its mean diameter \( d_{50}(j) \) and volume fraction in the mixture at every node \( \text{AVAI}(j) \). The characteristics of each class of sediment, such as the Shields number \( \theta_c(i) \) or the settling velocity \( w_s(i) \) for each class can be specified in the steering file or calculated by the model as for a single sediment class.

The percent of each class of material need to verify:

\[
\sum_{j=1}^{\text{NSICALA}} \text{AVAI}(j) = 1 \quad \text{and} \quad 0 \leq \text{AVAI}(j) \leq 1.
\] (47)

The initial bed composition can be defined in the steering file for uniform bed. For complex bed configurations, e.g. spatial variation, vertical structure, etc., the user subroutine \text{init_compo.f} is used to define the initial state. The mean diameter \( d_m, \text{ACLADM} \), is calculated as:

\[
d_m = \sum_{j=1}^{\text{NSICALA}} \text{AVAI}(j) d_{50}(j).
\] (48)

9.1.2 Bed structure

The bed is stratified into a number of layers \( \text{NOMBLAY} \) that enables vertical variations of the sediment bed composition. The percentage of each class \( j \) of material, \( \text{AVAI}(i,j,k) \) as well as the thickness of each layer \( E_s \) are defined at each point \( i \) and for each layer \( k \).

The composition of transported material is computed using the composition of the upper layer, see below the definition of active layer. The initial composition of the bed (number of layers, thickness of each layer \( E_s \), composition of each layer \( \text{AVAI} \) are specified in user’s subroutine \text{init_compo.f}.

The subroutine \text{init_avail.f} verifies if the structure of the initial bed composition is compatible with the position of the rigid bed, as defined in user’s subroutine \text{noerod.f}:

\[
Z_f(i) - Z_r(i) = \sum_{k=1}^{\text{NOMBLAY}} E_s(k).
\] (49)

In general, the thickness of the bed is taken to be large (by default, 100 m), so the bottom layer thickness of the last layer (nearest to the rigid bed) needs to be increased.

Assuming the porosity \( n \) of each class to be identical and constant, the total mass of sediments per unit area is:

\[
M_s(i) = \sum_{k=1}^{\text{NOMBLAY}} \rho_s E_s(k)(n - 1).
\] (50)

In each layer, the percent of each class \( \text{AVAI} \), which is defined as the volume of each class of material per the total volume of material, is considered to be a constant. For each layer and at each point of the domain, the following constraints need to be satisfied:
\[ \sum_{j=1}^{\text{NSICLA}} \text{AVAI}(i,k,j) = 1 \quad \text{and} \quad 0 \leq \text{AVAI}(i,k,j) \leq 1. \]

**Keywords**

The initial sediment bed composition is defined by:

- NUMBER OF SIZE-CLASSES OF BED MATERIAL: \( \text{NSICLA} = 1 \), by default
- SEDIMENT DIAMETERS: \( \text{FDM} = 0.01 \), by default
- SETTLING VELOCITIES: if \( \text{XWC} \) is not given, the subroutine \text{vitchu sisyphe.f} is used to compute the settling velocity based on the Stokes, Zanke or Van Rijn formulae as a function of the grain size
- SHIELDS PARAMETERS: for multi grain size, the Shields parameter needs to be specified for each class. If only one value is specified, the shields parameter will be considered constant for all classes. The default option, no Shields parameter given in steering file, is to calculate the Shields parameter as a function of the sand grain diameter, see logical \text{CALAC}.
- INITIAL FRACTION FOR PARTICULAR SIZE CLASS: \( \text{AVA0} = 1.;0.;0.;\ldots \), by default

For more than one-size classes, the user should specify\( \text{NSICLA} \) values for the mean diameter, initial fraction, etc. For example, for a mixture of two classes:

**Example**

- NUMBER OF SIZE-CLASSES OF BED MATERIAL = 2
- SEDIMENT DIAMETERS = 0.5; 0.5
- SETTLING VELOCITIES = 0.10; 0.05
- SHIELDS PARAMETERS = 0.045; 0.01
- INITIAL FRACTION FOR PARTICULAR SIZE CLASS = 0.5; 0.5

**Attention**

When using a (manually modified) sedimentological restart pay attention, that the following restrictions are valid:

\[ \sum_{j=1}^{\text{NSICLA}} \text{AVAI}(j) = 1 \quad \text{and} \quad 0 \leq \text{AVAI}(j) \leq 1 \quad (51) \]

\[ Z_f(i) - Z_t(i) = \sum_{k=1}^{\text{NOMBLAY}} E_s(k) \quad (52) \]

### 9.2 Sediment transport of sediment mixtures

The method programmed in \text{SISYPHE} for the treatment of mixtures of sediment with variable grain sizes is classical and based on previous models from the literature (as for example the 1D model Sedicoup developed at Sogreah). There are two layer concepts implemented in \text{SISYPHE}.

The classical active layer model based on Hirano’s concept \cite{23} and a newly developed continuous vertical sorting model (C-VSM), implemented by Merkel and Kopmann (BAW) \cite{36}.

#### 9.2.1 Active layer and stratum

Since only a relatively thin layer is involved in the transport, the upper part (active part) of the bed is therefore divided into a thin active layer and an active stratum. The composition of the active layer is used to calculate the composition of transported material and the intensity of transport rates.
for each class of sediment:

\[ Q_s = \sum_{j=1}^{NSICLA} AVAI(i, 1, j) Q_s(j), \]  

where \( AVAI(i, 1, j) \) is the percentage of the class \( j \) in the active layer.

The active stratum is the layer that will exchange sediment with the active layer in order to keep the active layer to a fixed (either constant or calculated) size. In case of high erosion depth, the active stratum becomes thinner and thinner, since it will be merged with the stratum lying underneath.

The active layer thickness concept is not well defined and depends on the flow and sediment transport characteristics [52]. According to Yalin [63], it is proportional to the sand diameter of the upper layer. The active layer thickness generally scales with the bed roughness, which is typically of the order of a few grain diameters for flat beds up to few centimeters in the case of rippled beds. In the presence of dunes, the active layer thickness should be half the dune height [39].

In SISYPHE, the active layer thickness is an additional parameter of the model, called ELAY0, which can be estimated by calibration and specified by the user in the steering file by keyword ACTIVE LAYER THICKNESS. With the option CONSTANT ACTIVE LAYER THICKNESS = NON, it is possible to use a space and time varying active layer thickness during the simulation. In SISYPHE it is assumed that \( ELAY0 = 3d_m \), with \( d_m \) the mean diameter of the active layer. Using the continuous vertical sorting model, more formulae for calculating the active layer thickness are available.

The erosion rate is restricted by the total amount of sediments in the active layer, which therefore acts as a rigid bed. The same method applied for rigid bed is applicable for the active layer formulation.

**Attention**

When dealing with graded sediment and thin thickness active layers, it is advised to use finite elements combined with the positive depth algorithm, as used for the treatment of rigid bed, in order to avoid numerical problems such as negative sediment fractions. The error message EROSION GREATER THAN ONE LAYER THICKNESS can also appear in the listing file when the erosion is greater than the active layer thickness. If the sedimentation is greater than the active layer thickness the error message DEPOSITION MORE THAN ONE LAYER THICKNESS appear in the listing as well. In both cases it is strongly recommended to reduce the time step. Otherwise the calculation of the fractions is not valid anymore.

### 9.2.2 Continuous vertical sorting model

The C-VSM overcomes many limitations of the classical layer implementation of Hirano (HR-VSM). Even though it is a different way to manage the grain sorting, it is just another interpretation of Hirano’s original idea with fewer simplifications. So still an active layer is defined but the grain distribution in this layer is computed for each time step from a depth dependent bookkeeping model for each grain size fraction with unlimited numerical resolution for each horizontal node. The new model does not overcome the need to carefully calibrate the same input parameters as all other models, but the new interpretation has the following advantages:

- It is possible to keep minor but prominent grain mixture variations even after a high number of time steps. Smearing effects and bookkeeping accuracy is defined by user defined thresholds or the computational resources, rather than through a fix value.
- Various functions for the impact depth of the shear stress can be chosen to the projects demands. The result is a much more accurate vertical grain sorting, which results in better prognoses for bed roughness, bed forms and erosion stability.

A couple of new keywords must be set in your SISYPHE steering file. The full C-VSM output can be found in the new Selafin files VSPRES & VSFHYD in the tmp-folders. As the higher resolution
of the C-VSM needs resources, you can reduce the printout period, or suppress the output at all. The common Sisyphē result files only show the averaged values for the active layer. Even more disk space can be saved, if only few points are printed out as .VSP .CSV files in the subfolder /VSP/. We recommend using between 200 and 1000 vertical sections. More will not improve the accuracy much, and less will lead to increasing data management, as the profile compression algorithms are called more often.

### Keywords

The initial sediment bed composition is defined by:
- VERTICAL GRAIN SORTING MODEL = 1
  - 0 = Layer = HR-VSM (Hirano + Ribberink as until Sisyphē v6p1)
  - 1 = C-VSM
- C-VSM MAXIMUM SECTIONS = 100
  - Should be at least $4 + 4 \times \text{Number of fractions}$, better > 100, tested up to 10000
- C-VSM FULL PRINTOUT PERIOD = 1000
  - 0 => GRAPHIC PRINTOUT PERIOD
  - Anything greater 0 => Sets an own printout period for the CVSP
  - Useful to save disk space
- C-VSM PRINTOUT SELECTION = 0|251|3514|...|...
  - Add any 2D Mesh Point numbers for .CSV-Ascii output of the CVSP
  - Add 0 for full CVSP output as Selafin3D files (called VSPRES + VSPHYD)
  - All files are saved to your working folder and in /VSP & /LAY folders below
- C-VSM DYNAMIC ALT MODEL = 5
  - Model for dynamic active layer thickness approximation
  - 0 = constant (Use Keyword : ACTIVE LAYER THICKNESS)
  - 1 = Hunziker & Guenther
    \[ ALT = 5d_{MAX} \]
  - 2 = Fredsoe & Deigaard (1992)
    \[ ALT = \frac{2\tauB}{(1-n)g(\rhoS - \rho)\tan\Phi} \]
  - 3 = van RIJN (1993)
    \[ ALT = 0.3D_{50}^{0.7}\frac{\tauB - \tauC}{\tauB} d_{50} \]
  - 4 = Wong (2006)
    \[ ALT = 5\frac{\tauB}{(\rhoS - \rho)g}\frac{0.0549}{0.56}d_{50} \]
  - 5 = Malcharek (2003)
    \[ ALT = \frac{d_{50}}{1-n} \max(1, \frac{\tauB}{\tauC}) \]
  - 6 = 3*50 within last time steps ALT'
    \[ ALT = 3d_{50} \]

#### 9.2.3 General formulation

Bedload transport rates are computed separately for each class using classical sediment transport formulae, corrected for sand grading effects. The Exner equation is then solved for each class of sediment. The individual bed evolution due to each class of bed material is then summed to give the global evolution due to bedload. Similarly, the suspended transport equation is solved for each class of sediment and the resulting bed evolution for each class is then summed to give the global
evolution due to the suspended load.

At each time step, the bed evolution due to bedload and to suspension transport is used to compute the new bed structure. The algorithm which determines the new bed composition considers two possibilities, namely deposit or erosion, and ensures mass conservation for each individual class of material. The algorithm is explained in [22].

9.2.4 Hiding exposure

In order to calculate bedload sediment transport rates for each class of sediment, it is necessary to consider the effect of hiding exposure. That means that in a sediment mixture, bigger particles will be more exposed to the flow than the smaller ones and therefore, prediction of sediment transport rates with classical sediment transport formulas, need to be corrected by use of a hiding-exposure correction factor. In Sisypho, the keyword HIDING FACTOR FOR PARTICULAR SIZE CLASS sets the value of hiding factor for a particular size class and the keyword HIDING FACTOR FORMULA allows the user to choose among different formulations. Two formulas, Egiazaroff [16] and Ashida & Michiue [1], have been written based on the Meyer-Peter and Muller formula. Both formulas modify the critical Shields parameter that will be used in the Meyer-Peter formula. Therefore they only can be combined useful with threshold formulations. The formula of Karim and Kennedy [30] can be used in combination with any bedload transport predictor. This formula modifies directly the bedload transport rate.

- HIDFAC = 1 : formulation of Egiazaroff
  \[ \theta_{cr} = 0.047 \xi_i, \quad \text{with } \xi_i = \left( \frac{\log(19)}{\log(D_i/D_m)} \right)^2 \]

- HIDFAC = 2 : formulation of Ashida & Michiue
- HIDFAC = 4 : formulation of Karim et al.

### Keywords

Sand grading effects are defined by:
- HIDING FACTOR FORMULA : if HIDFAC = 0 (by default), the user needs to give HIDING FACTOR FOR PARTICULAR SIZE CLASS
- ACTIVE LAYER THICKNESS : = 10000, by default
- CONSTANT ACTIVE LAYER THICKNESS : = YES, by default
- NUMBER OF BED LOAD MODEL LAYERS : NOMBLAY = 2, by default

9.2.5 Sediment transport formula

The formula of Hunziker [26] is an adaptation of the Meyer-Peter Muller formula for fractional transport. The volumetric sediment transport per sediment class is given by:

\[ \Phi_i = 5p_i \left( \xi_i (\mu \theta_i - \theta_{cm}) \right)^{3/2} \quad \text{if } \mu \theta_i > \theta_{cr}, \]  

(54)

with \( p_i \) the fraction of class \( i \) in the active layer, \( \theta_i \) the Shields parameter, \( \theta_{cm} = \theta_{cr} (D_{mo}/D_m)^{0.33} \) the corrected critical Shields parameter, \( \theta_{cr} \) the critical Shields parameter \( (\theta_{cr} = 0.047) \), \( \xi_i = (D_i/D_m)^{-a} \) the hiding factor, \( D_i \) the grain size of class \( i \), \( D_m \) the mean grain size of the surface layer (m), \( D_{mo} \) the mean grain size of the under layer (m), and \( a \) a constant. The critical Shields parameter is calculated as a function of the dimensionless mean grain size \( D_\ast \). It should be noted that according to Hunziker, stability problems may occur outside the parameter range \( a \leq 2.3 \) and \( D_i/D_m \geq 0.25 \). Hunziker developed an own hiding-exposure formula corresponding to his bed load formula.
Part III
Cohesive sediments
by Catherine Villaret, Lan Anh Van, Damien Pham van Bang & Pablo Tassi
10 Cohesive sediment properties

10.1 Introduction

Fine cohesive sediments are predominant in most estuaries, basins and reservoirs. The prediction of cohesive sediment transport, often associated with pollutants, is crucial regarding environmental issues.

Cohesive properties appear for fine particles (silts and clay), with diameter less than a limiting value of about 60 µm, depending on the physico-chemical properties of the fluid and salinity. Fine cohesive sediments are transported in suspension (no bed-load) and transport processes strongly depend on the state of floculation of the suspension and consolidation of the bed.

The erosion rate mainly depends on the degree of consolidation of the sediment bed, while the settling velocity depends on the state of floculation and aggregates properties.

**Attention**

The separation value at 60 µm to discriminate non-cohesive from cohesive sediment is conventional (UK classification). The value is different depending on the country (63 µm in Netherlands, 75 µm in USA as pointed by Winterwerp and Van Kesteren [61]). Moreover, aggregation of flocs can lead to the formation of macro-flocs larger than 100 µm.

**Keywords**

In **Sisyphe**, the simplest case of uniform and cohesive sediments is characterized by one single value for the primary grain size $D_{50}$ and density $\rho_s = 2650$ kg/m$^3$ for quartz particles, which is transported in suspension (no bedload).

In the **Sisyphe** steering file, the physical properties of the sediment are defined:

- **COHESIVE SEDIMENT** = YES (= NO, default option)
- **SEDIMENT DIAMETERS** ($D_{50} > 0.00006$ m, for non-cohesive sediment)
- **SEDIMENT DENSITY** ($\rho_s = 2650$ Kg/m$^3$, default value)

Both key words **SUSPENDED LOAD** = YES, **BED LOAD** = NO are set automatically, in order to be consistent with the selected type of sediment, when **COHESIVE SEDIMENT** = YES

- **SUSPENDED LOAD** = YES (= NO, default option)
- **BED LOAD** = NO (= YES, default option)

10.2 Effect of floculation

For cohesive sediments ($D_{50} < 60$ µm), the primary grain diameter and density no longer control the transport rates. Due to electro-static attractive forces (van der Waals), individual particles tend to form flocs. Their size can be an order of magnitude greater than the initial particle diameter, while the density decreases: the floc characteristics (size and density, as well as its fractal dimension) are more relevant to compute the settling velocity.

The settling velocity of cohesive particles therefore depends on the suspended sediment concentration as well as on other physico-chemical properties (pH, salinity, cations for instance) of the suspension which influence the flocculation process. The effect of turbulence level determines also the flocculation state. The critical bed shear strength depends on the consolidation state or age of the sediment bed (Migniot, 1968).

Flocculation can be modelled using process based models (cf. Mc Anally [34]). The settling velocity is generally one order of magnitude greater than the settling velocity based on the individual particle size, as a result of flocculation.

The flocculation process is not yet programmed in **Sisyphe** and the settling velocity should be specified by the user as a calibration parameter. More physically based relations will be implemented.
in a future release of the code.

### Keywords
- SETTLING VELOCITIES

**Attention**
The settling velocity needs to be specified, if not, it will be calculated by the Stokes law as a function of the individual particle diameter in subroutine `vitchu_sisyph.c`. As a result of flocculation, the settling velocity can be approximately an order of magnitude greater.

## 10.3 Consolidation process

Once the sedimentation process is achieved, a sediment bed is formed. For non-cohesive bed, no evolution with time will be observed if no erosion or further sedimentation occurs. For cohesive bed, the concentration will increase with time as the result of self-weight consolidation or compaction. Two stages, namely the primary and secondary consolidation are classically considered to explain the progressive compaction of the cohesive sediment bed with time. As the formed bed resulting from the sedimentation of mud is very soft, it tends to compact under its self-weight. During this process, the void ratio tends to diminish so that pore water is expelled upward as a result of water incompressibility. The dynamic of this primary stage of consolidation is therefore controlled by the permeability of the bed. At the end of the primary consolidation, the excess pore pressure is fully dissipated (Figure 2) but the cohesive bed records further compaction. This secondary stage of consolidation is no longer related to pore pressure but rather to the solid skeleton property. The secondary consolidation is considered as a result of time effect (viscous or ageing) in the stress-strain relationship of the soil. Two effects of time are currently considered in modern soil mechanics: the creep of the soil (viscous behaviour of cohesive geomaterial) and ageing (thixotropy of mud). Even through the dynamic of the secondary consolidation is much slower than the primary consolidation, this stage is important for long term prediction.

### Keywords
- MUD CONSOLIDATION = NON, by default

## 11 Cohesive bed structure

### 11.1 Multi-layer model

Cohesive sediment beds are generally non-uniform: as a result of self-weight consolidation, the bed becomes stratified, with density increasing with distance from the surface. The top layer is generally made of freshly deposited soft mud, while the bottom consolidated layers present higher resistance to erosion. The vertical stratification of the cohesive bed is therefore a key issue, which controls the amount of material to be put in suspension.

In SISYPHE, cohesive sediment bed can be represented by a fixed number `NCOUCH_TASS` of layers. The number of layers is fixed with maximum number up to 20. Each layer is characterized by its concentration and resistance to erosion. The concentration of each layer \(C_s(j)\) is generally constant and specified by keyword `MUD CONCENTRATION PER LAYER` expressed in kg/m³. It may also vary in space and time (see consolidation model 3). The resistance of each layer is specified by keyword `CRITICAL EROSION SHEAR STRESS OF THE MUD` expressed in N/m².

Each layer \(j\) (\(1 \leq j \leq NCOUCH_TASS\)) is defined by its concentration \(C_s\), thickness \(E_s\) and mass
Figure 2 – Vertical variation of density and effective stress within the cohesive sediment bed.

per surface area $M_s$, which depends on both layer concentration and thickness, such that

$$M_s(i,j) = C_s(j)E_s(i,j)$$

where $M_s$ is the mass per unit surface area (Kg/m$^2$), $C_s$ is the concentration (kg/m$^3$) and $E_s$ the layer thickness.

For specific applications, the bed concentrations $C_s(i,j)$ can also vary from one point to another. The initial distribution can be specified in subroutine *init_compo_coh.f*.

**Keywords**

- **NUMBER OF LAYERS OF THE CONSOLIDATION MODEL**: *NCOUCH_TASS* (default value = 1) should be less than the maximum number of layers (*NLAYMAX* = 20)
- **MUD CONCENTRATION PER LAYER**, in Kg/m$^3$: $C_s$ (default value = 50)

**Attention**

The keywords **NUMBER OF BED LOAD MODEL LAYERS** and **NUMBER OF LAYERS OF THE CONSOLIDATION MODEL** are essentially the same except that the default values are different.

For cohesive sediments, it is possible to have only one uniform layer, whereas for sand grading algorithm we need at least 2 layers (the active layer and the substratum). The subroutine *lecdon.f* specifies at the end $NOMBLAY = NCOUCH_TASS$.

**Attention**

Due to the default value of $NOMBLAY$ for sand grading effects ($NOMBLAY = 2$). For uniform beds both keywords need to be specified:

- **NUMBER OF BED LOAD MODEL LAYERS** = 1
- **NUMBER OF LAYERS OF THE CONSOLIDATION MODEL** = 1
11.2 Initialization

The initial concentration for the suspended load can be either imposed within condim_susp.f or specified in the steering file through the keyword INITIAL SUSPENSION CONCENTRATIONS initializes the value of the volume concentration for each class.

The initial layer thicknesses are specified in user-subroutine init_compo_coh.f. It is called by subroutine init_mixte.f which just checks if the sum of all layers is equal to the total bed thickness as specified by the initial bathymetry $Z_f$ and rigid bed $Z_r$:

$$ Z_f(i) - Z_r(i) = \sum_{j=1, \text{NCOUCH,TASS}}^{\text{NCOUCH,TASS}} E_j(i,j) $$  \hspace{1cm} (55)

where $i$ is the number of point, $j$ the number of layer, $Z_f$ the bed level and $Z_r$ the non-erodible bed level.
12 Erosion and deposition rates

The erosion and deposition rates are needed to solve the net sediment flux present in the advection-diffusion equation 34, see Chapter 6. In this equation, the erosion term is treated explicitly while the deposition term is treated implicitly.

Attention
The sediment concentration is assumed to be uniform over the depth, which is a reasonable assumption for fine cohesive sediments ($w_s << u^*$).

12.1 Erosion flux

Erosion fluxes can be calculated using the Partheniades classical formula which is programmed in Sisyphé: erosion flux are related to the excess of applied bed shear stress to the bed shear strength at the bed surface. When the bed is stratified, it is necessary to take into account the vertical increase of bed shear strength as the bed gets eroded.

Uniform bed

The classical Partheniades formula is applied for cohesive sediments. Assuming the bed to be uniform ($NCOUCH_TASS = 1$):

\[
E = \begin{cases} 
M \left( \left( \frac{u_s}{u_{se}} \right)^2 - 1 \right), & \text{if } u_s > u_{se} \\
0, & \text{otherwise}
\end{cases}
\]

where $u_{se}$ is the critical erosion velocity and $u_s$ is the friction velocity related to skin friction, defined as:

\[
u_s = \sqrt{\frac{\tau'}{\rho}}
\]

with $\tau'$ the bed shear stress corrected for skin friction and $\rho$ the fluid density.

The Partheniades coefficient $M$ is a dimensional coefficient. It is specified in the steering file with units given in kg/m$^2$/s. The erosion rate is expressed in m/s, by converting the $M$ coefficient to m/s (subroutine lecdon.f).

Non uniform bed

In the multi-layer model, the critical erosion shear stress increases as the mud concentration increases (see Migniot, 1968).

In the multi-layer model ($NCOUCH_TASS > 1$), each layer is characterized by its density and critical bed shear stress. The erosion rate of each layer $E(j)$ decreases with distance from the surface.

For each layer $j$ and at each time step, the erosion rate $E(j)$ is calculated as a function of the difference between the applied bed shear stress and the critical bed shear strength, using (4). For each $E(j) > 0$, the layer is erodible.

The erosion flux $E$ is determined as the mean erosion rate, averaged over the eroded depth:

\[
E = \frac{1}{\Delta z_{er}} \int_0^{\Delta z_{er}} E(z)dz = \frac{1}{\rho_s dt} \int_0^{dt} dM_s
\]

where $dM_s$ is the eroded mass per unit surface area (Kg/m$^2$), $\Delta z_{er}$ is the maximum depth to be eroded, $E(z)$ is the erosion flux (m/s) at distance $z$ from the bed interface, and $dt$ the time step.
Iterative procedure

The potential depth to be eroded is estimated using the iterative procedure described below and programmed in subroutine suspension_erosion_coh.f.

At each time step, the top layer is first eroded if $E(1) > 0$. Once it is empty, the next layer is eroded etc... For each erodible layer $j$ with erosion rate ($E(j) > 0$), the time interval $dt(j)$ to erode it completely is estimated by a simple mass balance from the mass of the layer $M_s(j)$:

$$dt(j) = \frac{M_s(j)}{\rho_s E(j)} = \frac{E_s(j)C_s(j)}{\rho_s E(j)}$$

For the first layer ($j = 1$), there are two possibilities:

- $dt(1) > dt$: layer 1 is emptied partially, such that $-C_s(1)dE_s(1) = \rho_s dt(j)$
- $dt(1) < dt$: layer 1 is emptied completely, and the next (second) layer can be eroded, such that $dE_s(1) = -E_s(1)$.

The last (non-empty) layer (noted $j_{\text{max}}$) to be eroded is obtained by emptying all successive top layers until:

$$\sum_{j=1}^{j_{\text{max}}-1} dt(j) < dt < \sum_{j=1}^{j_{\text{max}}} dt(j)$$

For $j_{\text{max}}$, the time left to erode the last layer is

$$dt_{\text{max}} = dt - \sum_{j=1}^{j_{\text{max}}-1} dt(j)$$

The mass potentially eroded during this process is

$$\overline{M}_s = \sum_{j=1}^{j_{\text{max}}-1} M_s(j) + \rho_s E(j_{\text{max}})dt_{\text{max}}$$

The erosion flux (m/s) is therefore estimated as follows:

$$E = \frac{\overline{M}_s}{\rho_s dt}$$

Keywords

- COHESIVE SEDIMENTS (= YES, default option)
- PARTHENIADES CONSTANT ($M = 10^{-3}$ Kg/m$^2$, default option)
- CRITICAL EROSION SHEAR STRESS OF THE MUD ($\tau_{ce} = 0.01$ N/m$^2$, default option)

12.2 Deposition flux

The deposition flux is calculated as a function of the near bed concentration. In the case of fine cohesive sediments ($w_s << u_*$ where $w_s$ is the settling velocity, and $u_*$ the turbulent friction velocity), the bed concentration is approximately equal to the depth-averaged concentration. The deposition flux can be treated as an implicit term, whereas the erosion flux is explicit.

$$D = \begin{cases} 
0 & \text{if } u_* > u_{sd} \\
\frac{w_s C}{1 - \left(\frac{u_*}{u_{sd}}\right)^2} & \text{otherwise}
\end{cases}$$
where \( C \) is the depth-averaged concentration, \( w_s \), the settling velocity, and \( u_{*d} \), the critical deposition velocity which represents the limiting shear velocity, above which the sediment flocs are broken and resuspended.

### Keywords
- CRITICAL SHEAR VELOCITY FOR MUD DEPOSITION \((u_{*d} = 1000 \text{ m/s, default option})\)
- SETTLING VELOCITIES (by default, \( w_s \) is calculated by the model based on the Stokes law and individual particle diameter)

## 13 Bed Evolution

The bed evolution for cohesive sediments is solved with the Equation (43), see Chapter 6.

### Uniform bed

The uniform bed \((NCOUCH,TASS = 1)\) is characterized by a single density \( C_s = C_s(1) \). The bed evolution at each time step is therefore:

\[
dZ_f = \frac{\rho_s(D - E)dt}{C_s} \]

\((dZ_f < 0 \text{ for net erosion and } dZ_f > 0 \text{ for net deposition}).\)

### Non-uniform beds

For non-uniform beds \((NCOUCH,TASS > 1)\), two different cases occur.

- In the case of net deposition \((D - E > 0)\), sediment is deposited in the first top layer of density \( C_s(1) \):

\[
dZ_f = dE_s(1) = \frac{\rho_s(D - E)dt}{C_s(1)} > 0
\]

- In the case of net erosion \((E - D > 0)\), sediment is eroded layer by layer \((dE_s(j) < 0)\), until the eroded mass balances the net eroded flux. The last layer to be eroded \( j_{\text{max}} \) is determined in order to satisfy:

\[
\sum_{j=1}^{j_{\text{max}}-1} M^n_s(j) < \rho_s(E - D)dt < \sum_{j=1}^{j_{\text{max}}} M^n_s(j)
\]

Top layers are emptied \((dE_s(j) = -E_s(j) \text{ for } j = 1 \text{ to } j_{\text{max}} - 1)\); the last layer is eroded up to a the depth \( dE_s(j_{\text{max}}) < 0 \) in order to satisfy mass conservation:

\[
\sum_{j=1}^{j_{\text{max}}-1} M^n_s(j) - dE_s(j_{\text{max}})C_s(j_{\text{max}}) = \rho_s(E - D)dt
\]

The variation of bed elevation is therefore:

\[
dZ_f = -\left( \sum_{j=1}^{j_{\text{max}}-1} E^n_s(j) \right) + dE_s(j_{\text{max}}) < 0
\]

The thickness and mass of each layer are updated at the end of each time step \((n + 1)\), from their initial values \((n)\):

\[
\text{for } j = (1, j_{\text{max}} - 1) \quad \begin{cases} E^{n+1}_s(j) = 0 \\ M^{n+1}_s(j) = 0 \end{cases}
\]
and

\[
\begin{align*}
E_n^{j+1}(j_{\text{max}}) &= E_n^j(j_{\text{max}}) + dE_n(j_{\text{max}}) \\
M_s^{j+1}(j_{\text{max}}) &= E_n^{j+1}(j_{\text{max}})C_s(j_{\text{max}})
\end{align*}
\]

Underneath layers (from \(j_{\text{max}} + 1\) to \(\text{NCOUCH}\_\text{TASS}\)) are not eroded and keep their thickness and mass.

## 14 Mass conservation

The subroutine \texttt{suspension\_bilan\_coh.f} calculates at each time step the mass of sediments in the computational domain and ensures that the sum of the different components is compensated by the fluxes at the liquid boundaries. The mass of sediment in suspension \(M_1\) is given by:

\[
M_1 = \rho_s \iiint c(x,y,z) \delta v = \rho_s \int S C(x,y) h \delta s
\]

with \(C\) the depth-averaged volume concentration of sediment in suspension, \(h\) the water depth and \(S\) the surface of the computational domain.

In finite elements, each 2D variable is decomposed on basis function \(\phi_i\):

\[
M_1 = \rho_s \sum_{i=1,N\text{poin}} C_i h_i \int \phi_i \delta s = \rho_s \sum_{i=1,N\text{poin}} C_i h_i S_i
\]

with \(S_i\) the integral of basis function. The mass of sediment in the bed \(M_2\) is expressed as:

\[
M_2 = \iiint C_s \delta v = \int \left( \int_{Z_f}^{Z_r} C_s \delta z \right) \delta s
\]

with \(C_s\) the sediment bed mass concentration, \(Z_f\) the bed level and \(Z_r\) the non erodible bed level. After discretization of the bed layers into layers of variable thickness \(E_s\) and concentration \(C_s\)

\[
M_2 = \iint \left( \sum_{j=1,N\text{omblay}} M_s(j) \right) \delta s = \sum_{i=1,N\text{poin}} \left( \sum_{j=1,N\text{omblay}} M_s(j) \right) S_i
\]

where \(M_{si}\) is the total mass per surface area at node \(i\).

**Keywords**

- MASS-BALANCE (= NON, default option)

## 15 Consolidation algorithm

In Sisyphé, the effect of consolidation is modeled through physical as well as empirically-based models. In a so-called iso-pycnal scheme, the bed is represented by a number of layers of increasing concentration. The concentration of each layer is fixed, while their thickness varies in time as the bed undergoes erosion/deposition/consolidation. This scheme is used in a semi-empirical model, originally developed by Villaret and Walther [58]. A new scheme developed by Van [35] is based on the Gibson’s theory. An other third scheme has been implemented which is similar to the Gibson consolidation model developed in \\textsc{Telemac-3d} by LeNormand et al. [32]. In this third model, the bed is discretised in layers of given thicknesses and time-varying concentrations. This third model has been shown to be unstable and CPU time-consuming (cf. [35]).
Consolidation theory

1. Most consolidation theories describe the primary consolidation since they are based on the theory by Terzaghi (1923) [50].

2. The theory by Terzaghi relates the deformation of the bed to the permeability and the effective (or solid) stress which is defined as the difference between the total stress and the pore pressure (principle of effective stress [50]). It corresponds to the stress that is transmitted directly through the contacts between solid particles. It is also called osmotic pressure by some authors. Been and Sills (1981) evidenced the progressive increase of effective stress during the consolidation process.

3. The theory by Terzaghi was originally formulated for infinitesimal strains so that permeability and compressibility could be assumed constant. The theory was extended for large deformations by Gibson et al. [20, 21] as pointed out by Been and Sills [6].

As an illustration of the process in the water column, Figure 3 proposes a schematic representation of both, the sedimentation and the consolidation. In the right side of Figure 3, we present the validity range of the Kynch theory of sedimentation and of the Gibson theory of large strain consolidation. Both theories are unified by Toorman [47, 48] which pointed out the fact that Gibson model can also describe the sedimentation of particle depending on the choice of closure equations.

The general model (Equations 56, 57 and 58) proposed by Gibson et al. [20, 21] represents the different stages of consolidation. This equation is based on a two-phase approach by considering continuity and motion equations for both fluid and solid phase to obtain the general equation that reads in material coordinate $\zeta$ which represents the volume of solids:

$$\frac{\partial \varepsilon}{\partial t} + \left(\frac{\rho_s - \rho_f}{\rho_f}\right) \frac{d}{d\varepsilon} \left(\frac{k}{1+\varepsilon}\right) \frac{\partial \varepsilon}{\partial \zeta} + \frac{\partial}{\partial \zeta} \left(\frac{k}{g\rho_f(1+\varepsilon)} \frac{\partial \sigma'}{\partial \varepsilon} \frac{\partial \varepsilon}{\partial \zeta}\right) = 0,$$

where $\varepsilon$ is the void ratio, $k$ the hydraulic permeability (in m/s) and $\sigma'$ the effective (or solid) stress.

The Gibson equation can be also written in Eulerian framework as:

$$\frac{\partial \varepsilon}{\partial t} + (1+\varepsilon)^2 \left(\frac{\rho_s - \rho_f}{\rho_f}\right) \frac{\partial}{\partial z} \left(\frac{k}{(1+\varepsilon)^2}\right) + \frac{(1+\varepsilon)^2}{g\rho_f} \frac{\partial}{\partial z} \left(\frac{k}{1+\varepsilon} \frac{\partial \sigma'}{\partial \varepsilon}\right) = 0$$

This equation is equivalent to:

$$\frac{\partial \phi}{\partial t} - \frac{\partial}{\partial z} \left[\left(k(s-1)\phi + \frac{k}{\gamma_f} \frac{\partial \sigma'}{\partial z}\phi\right)\right] = 0$$

where $\phi$ stands for the sediment volume concentration, $k$ the hydraulic permeability, $s$ the density ratio between sediment and fluid ($=\rho_s/\rho_f$), $\gamma_f$ the unit weight of fluid ($=g\rho_f$, $g$ being the acceleration of gravity), $z$ the vertical coordinate (positive upward), and $t$ the time.

The equation of Gibson has been widely used in various numerical consolidation models [3, 6, 47, 48] as well as compared with the experimental results. It has been implemented in TELEMAC-3D by LeNormand et al. [32] or coupled with TELEMAC-2D by Thiébot [45]. Their method of resolution has been adapted to SISYPHE by Van [35].

The main difficulty in using the Gibson model is related to the choice for closing the problem. Two closure equations, for the permeability $k$ and for the effective stress $\sigma'$, is indeed required to obtain the time evolution of vertical concentration profiles. However, the formulation of these closure equations remains an open problem and a shared protocol to determine their parameter values are still lacking as reported by Toorman [47, 48], Bartholomeeusen et al. [3] and Van [35].
15.1 Multi-layer empirical algorithm

Time evolution

The consolidation effect is reproduced by assuming that the vertical flux of sediment from layer $j$ to underneath layer $j+1$ is proportional to the mass of sediments, $M_s$ (kg/m$^2$), contained in the layer $j$.

$$\frac{dM_s(j)}{dt} = a_i M_s(j) \quad (59)$$

The transfer mass coefficients $a_i$ (in s$^{-1}$) are specified in the steering file (MASS TRANSFER PER LAYER). They correspond to a characteristic timescale to transfer mass from one layer to another.

This multilayer consolidation model is not related to Gibson equation so that it should be considered as empirical. Despite its apparent simplicity, it can qualitatively reproduce an increase of mud bed deposit with time, while ensuring mass conservation (transfer coefficient of the last bottom layer is zero). The model results are highly sensitive to the specified values of the mass transfer coefficients $a_i$ which are however difficult to calibrate. The main advantage of this iso-pycnal formulation relies in the fact that no vertical grid is required to compute the concentration evolution of the different layers.

Discretization

The vertical resolution of semi-empirical equation (59) is based on the multi-layer iso-pycnal model with fixed concentrations. The consolidation is then reproduced by mass transfer between layers of the model. The transfer coefficients are fixed for each layer.

The set of mass-transfer coefficients $a(j)$ in (s$^{-1}$) are selected by calibration, in order to find best agreement of time-varying concentration profiles between model and experiment. Physically, they represent the inverse of the residence time per layer. The values $a(j)$ are found to decrease from top to bottom, as the time scale of consolidation (or residence time) increases. The mass transfer of the last layer is set to zero, in order to insure no mass loss at the rigid bed level (impermeability condition).
Keywords

- COHESIVE SEDIMENT = YES (= NO, default option)
- MUD CONSOLIDATION = YES (= NO, default option)
- CONSOLIDATION MODEL = 1, default option
- NUMBER OF LAYERS OF THE CONSOLIDATION MODEL (NOMBLAY=10 by default, maximum value is 20)
- MUD CONCENTRATION PER LAYER (in kg/m$^3$, E = 50.; 100.; ... by default)
- MASS TRANSFER PER LAYER (= 5.0E−5, ..., 0., by default)

15.2 Multi-layer iso-pycnal Gibson’s model

15.2.1 Theoretical background

The previous equation presents the advantage of not using a vertical mesh. However it is not physically based model since it neglects the Gibson theory. Improvement of this category of model was proposed by Sanchez [41] and Thiebot [45]. In these formulations, the sediment flux from one layer to the other is given not in term of empirical mass transfer coefficient ($a_i$) but in term of theoretical flux which is given by the Gibson theory.

15.2.2 Numerical discretization

The model originally developed by Sanchez [41] and Thiebot [45] is a 1DV sedimentation-consolidation multi-layer model, based on an original technique to solve Gibson equation. The advantage of this representation (Equation 60) if compared with previous one (Equation 59) relies on the right hand side term which relates the mass of sediments, $M_S(i)$ (kg), to the net flux entering or leaving the layer. This equation enables therefore to consider the flux of sedimentation and consolidation as provided by the Gibson theory.

$$\frac{dM_S_i}{dt} = (F_i(t) - F_{i+1}(t)) \Delta t \pi r^2$$ (60)

In Equation 60, a circular shape (of radius $r$) is assumed for the surface. Equivalently, Equation 61 is expressed in term of layer thickness $E_S(i)$ (m), recalling $M_S = C_S \pi r^2 E_S(i)$. This last expression becomes independent on the settling column geometry.

$$\frac{dE_S_i}{dt} = \frac{F_i(t) - F_{i+1}(t)}{C_S}$$ (61)

The concentration of the different bed layers $C_S(i)$ is fixed. As the sedimentation and consolidation progress, the sediment is transferred to the more concentrated layers, and the thickness of these layers increase as well.
The mass conservation is ensured by requiring at each moment, in each layer, the equality between the mass contained in a layer at time \( t + \Delta t \) and the mass present in this layer at time \( t \) in which the outgoing mass was removed and the incoming mass was added (means the mass that crossed the upper and lower sections respectively during the time \( \Delta t \)). The outgoing and incoming masses are taken into account by sediment flux noted \( F_i(t) \).

\[
F_i(t) = \frac{(V_{s,i}(t) - V_{s,i-1}(t))C_{i-1}C_i}{C_i - C_{i-1}} \quad (62)
\]

where \( V_{s,i} \) is the falling velocity of the layer \( i \), and can be defined as:

\[
V_{s,i}(C_i) = \begin{cases} 
  k(C_i)C_i \left( \frac{1}{\rho_s} - \frac{1}{\rho_f} \right), & \text{if } C_i \leq C_{gel} \\
  k(C_i)C_i \left( \frac{1}{\rho_s} - \frac{1}{\rho_f} \right) + k(C_i) \frac{\sigma'(C_{i-1}) - \sigma'(C_i)}{2(Ep_{i-1}(t) + Ep_i(t))}, & \text{otherwise} 
\end{cases}
\]

where \( k \) is the permeability, \( \sigma' \) is the effective stress, \( C_{gel} \) the transition concentration between sedimentation and consolidation schemes (see Camenen & Pham Van Bang [9]).

### 15.3 Vertical grid Gibson’s model

Vertical-grid models propose a natural resolution of the Gibson equation as a vertical grid is used to compute the concentration at each point. This category of model uses common techniques (finite difference or finite volume or finite element methods) for resolving partial differential equations.

As they use a vertical grid, the connection with SISYPHE which is depth integrated model is not straightforward.

In the new version of SISYPHE, such a category of model is also available. Strictly speaking, the original Gibson model (in material coordinate) used in TELEMAC-3D which was developed by Lenormant et al. [32] is connected to SISYPHE.

The finite difference method is used. The implicit scheme leads to a tridiagonal matrix that is solved by using a classical double sweep algorithm. More details are given in TELEMAC-3D user guide.
guide, LeNormand et al. [32] and Van [35].

16 Closure equations for permeability and effective stress

All of the Gibson based models, i.e. Multi-layer iso-pycknal Gibson’s model (section 15.2) and vertical grid Gibson model (section 15.3), requires two closure equations. The Multi-layer empirical model (section 15.1) only needs closure on ai, the mass transfer coefficients.

In this section, some available empirical formula are detailed. In general, a decreasing (or increasing) function of solid concentration is considered for permeability $K$ (or effective solid stress). These expressions are logarithmic or exponential or power law functions (see Bartholomeeusen et al. [3]).

16.0.1 Closure equations

- **Bed permeability $k$**
  Bartholomeeusen et al. [3] introduced typical functions, in the form of either power or exponential, to relate the permeability $k$ with the void ratio $e$:

  $$k = \begin{cases} A_1 e^{A_2} \\ A_1 \phi_s^{-A_2} \\ \exp(-A_1 + A_2 e) \end{cases}$$

  The value of these coefficients $A_1$ and $A_2$ depend on grain size distribution, organic content, activity and pore size distribution.

- **Effective stress $\sigma'$**
  The similar way can be opted in the determination of $\sigma'(e)$ ($\sigma'(C), \sigma'(\phi)$), which gives:

  $$\begin{cases} e = -B_1 \sigma'^{B_2} + B_3 \\ \sigma' = B_1 \phi_s^{B_2} \end{cases}$$

  or

  $$\begin{cases} e = B_1 (\sigma' + B_2)^{-B_3} \\ \sigma' = \exp(B_4 + B_5 e) \end{cases}$$

  According to Winterwerp & van Kesteren (2004), the validity range of the power-type functions (e.g. $k = A_1 e^{A_2}, \sigma' = B_1 \phi_s^{B_2}$) is much larger than that of the exponential-type functions. Moreover, there is a physical insight in the formulation of the power-type functions. Therefore, it is recommended to use power-type functions. However, two different power functions for permeability may be necessary to represent the two separate processes: sedimentation and consolidation.

16.0.2 Determination of parameters

No standard is found in the literature to recommend a specific methodology for calibrating the empirical functions for both permeability and effective stress. Most of study reported some fitting exercize on settling curve, i.e. the position of supernatant/suspension interface. They considered mostly the least square technique for the adjustment to experimental results. However as stated by Toorman [48] a more relevant adjustment is offered when concentration profiles are available from Gamma-ray techniques (see Been and Sills [6] or Bartholomeeusen et al. [3]), X-ray technique (Villaret et al. [59]) or MRI techniques (Pham Van Bang et al. [38]). In such a situation, the density or concentration profile are adjusted on the measurement for different time. A specific procedure has been recently proposed by Thiebot et al. [46], also based on least square method.

The user should consider the previous method (adjustment on settling curve or on concentration profile) as conventional even through no shared procedure has been internationaly recognized as the best one (as previously discussed in 10.3). An recent alternative was successfully applied to Gironde mud (Van [35]).
16.0.3 Space-time based method to determine parameters of closure equations

The new methodology to determine parameters is based on space-time analysis of data. It requires data on time evolution of concentration (vertical) profile. If such an information is not available, the proposed method becomes useless. If only the supernatant/suspension interface position is detailed by data, the user should consider the classical fitting method.

If the space-time resolved data is available, this new methodology could be applied easily. The method is based on a theoretical framework which is an advantage to help during the calibration procedure. From time evolution of concentration profiles which are provided by non-intrusive techniques (here the X-ray technique), the procedure uses self-similar analytical solutions to determine the closure equation relative to the convection (or sedimentation) part and to the diffusion (or consolidation) part of Gibson equation.

\[
\frac{\partial \phi}{\partial t} - \frac{\partial}{\partial z} \left[ V(\phi) \phi \right] - \frac{\partial}{\partial z} \left[ D(\phi) \frac{\partial \phi}{\partial z} \right] = 0
\]

where \( V(\phi) = K(s-1)\phi \) and \( D(\phi) = K\phi/\gamma_f d\sigma'/d\phi \) in order to match with Gibson equation 58 in Eulerian coordinate.

Considering a separation regime between sedimentation and consolidation (or between convection and diffusion), self-similar solutions are obtained for each regime. The separation between both problems is justified by the fact that effective stress should be zero for a suspension since there is no direct contact between particles.

16.0.4 Determination of closure equation for the sedimentation

If the concentration of the suspension is lower than a given threshold (the so-called gelling point for cohesive sediment), the inter-particle contacts are negligible, i.e. there is no solid (or effective) stress (Camenen & Pham Van Bang [9]). In such a situation the Gibson equation (58) is simply reduced to the equation of Kynch :

\[
\frac{\partial \phi}{\partial t} - \frac{\partial}{\partial z} \left[ V(\phi) \phi \right] = 0
\]

where \( \phi = C/\rho_s \) is the volume fraction of solids, \( V(\phi) \) is the settling velocity of the suspension at concentration \( \phi \) that is equal to \( K(s-1)\phi \).

Considering the self-similar variable \( \zeta = z/t \) and similarity solution \( U \), i.e. \( \phi(z,t) = U(\zeta) \), equation 64 leads to :

\[
\left( \frac{d}{dU} - \zeta \right) \frac{dU}{d\zeta} = 0
\]

where \( f \) is the solid (or sedimentation) flux, which is equal to \( -V(\phi)\phi \).

The method is equivalent to the so-called method of characteristics (Leveque [33]). The iso-concentration pattern presents different straight lines in the space-time \((z-t)\) plot. The slopes of iso-concentration straight lines in the \( z-t \) plane are measured to obtain the first derivative of the solid flux. The sedimentation flux proposed for the Gironde mud (Villaret et al. [59]; Van [35]) is given by equations 65 or 66. The first derivative of this closure equation is straightforward [59] : the determination and validation of its parameters \( (V_{st}, \phi_{gel}, n) \) has been presented in details in the test case of settling column of Gironde mud :

\[
f(\phi) = V_{st}(1-\phi) \left( 1 - \frac{\phi}{\phi_{gel}} \right)^n \phi \quad \text{for} \quad \phi < \phi_{gel}
\]
16.0.5 Determination of closure equation for the consolidation

Still considering separation regime between sedimentation (convective problem) and consolidation (diffusion problem), for cohesive sediment and concentration larger than the gelling point, the effective solid stress build up. The diffusion term is no longer negligible and becomes the leading term in the Gibson equation. For concentration larger than the gel point, the convective part is annihilated in the proposed formulation so that only the diffusion term remains. As this term takes the form of a power law, i.e. $D = D(C) = C^n$, the diffusion equation is expressed as:

$$ f(C) = \frac{V_{st} (1 - \frac{C}{\rho_s}) \left( 1 - \frac{C}{C_{gel}} \right)^n \frac{C}{\rho_s}}{\eta_s} \quad \text{for} \quad C < C_{gel} \\ (66) $$

where $V_{st}$ is the Stokes velocity of an equivalent sphere, $C_{gel}$ is the gelling concentration, $n$ is an exponent. Both parameters have physical meanings in terms of rheology (Pham Van Bang et al. [38]). Indeed, $f(C)$ is the concentration value from which the effective viscosity of the suspension diverges. And $n$ is a parameter describing the transition from suspension to a structured bed. It is worth noting that the so-called Richardson & Zaki empirical model is recovered by setting $f(C) = 1$.

### 16.0.5 Determination of closure equation for the consolidation

The parameter $M$ is a constant whose value corresponds to the total mass of sediment in the system.

In order to find out the closure equation for effective stress, from the above equations, it is needed to determine the two variables : $a$, $b$. This procedure will be presented in detail in test case Tassement_2, with a given experimental result of settling column.
### Keywords

- **COHESIVE SEDIMENT** = YES (= NO, default option)
- **MUD CONSOLIDATION** = YES (= NO, default option)
- **CONSOLIDATION MODEL** = 2
- **GEL CONCENTRATION** = CONC_GEL
- **PERMEABILITY COEFFICIENT** = COEF_N
- **NUMBER OF LAYERS OF THE CONSOLIDATION MODEL** = NOMBLAY (< 20)
- **MUD CONCENTRATION PER LAYER**
17 Mixed sediments processes

17.1 Mixte sediment bed composition

A sand-mud mixture is composed of two classes of bed sediments: The first class (noted 1) is non-cohesive and represented by its grain diameter $D_1$ and can be transported as bed-load or suspended load. The settling velocity $w_s^1$ is a function of the relative sediment density ($s = 1.65$) and grain diameter $D_1$. So far we assume only suspended load, which implies that the model is devoted to mixtures of fine sand grains and mud.

The second class (noted 2) is cohesive, grain diameter $D_2$ less than 60 mm. The settling velocity $w_s^2$ is a function of flocs properties which differs from the individual cohesive particles, and needs to be specified. The mass concentration is the mass of sediments per volume occupied by the mud component (solid particles + water).

17.2 Erosion/deposition of sand-mud sediment bed

The erosion/deposition fluxes depend on the mass percentage % of each class in the surface layer (Panagiotopoulos et al., 1997). If the mass percent of the mud class is greater than 50%, the bed is considered as pure cohesive and the erosion/deposition laws follow the Partheniades classical law (see below for the calculation of the bed shear strength in a sand/mud mixture).

If the mass fraction of mud percentage is less than 30%, the bed is considered as non-cohesive, and has little effect on the bed shear strength. If the mass fraction of mud percent is greater than 30%, the bed is considered as cohesive. The rate of erosion is based on the Partheniades erosion law, where the critical bed shear strength of the mud class depends on the consolidation state. Different regimes can be considered depending on the volume content of each class, as pictured below:

![Figure 5](image)

**Figure 5** – (a) Sand regime ($f_2 < 30\%$); (b) Intermediate; (c) Mud-dominated regime ($f_2 > 50\%$).

17.3 Critical erosion shear stress of sand mud mixture

The mean bed shear strength ($\tau_{ce}$) of the sand/mud mixtures depends on the mass percentage % of mud $f_2$ at the surface and depends on the layer. The proposed framework is based on the work of Waeles (2005). The bed shear strength is calculated in the subroutine `suspension_flux_mixte.f`
(TOCE_MIXTE, for \( \tau_{ce} \)). For each layer \((K)\), the expression of TOCE_MIXTE depends on the mass percent of mud in the layer:
- For \( f_2 < 30\% \), \( \tau_{ce} = \tau_{ce}(1) \) for pure sand, based on
  \[ \tau_{ce} = \tau_{ce}(1) = \Theta c g d_{50} (s - 1) \]  
  \( (68) \)
- For \( f_2 > 50\% \), \( \tau_{ce} = \tau_{ce}(2) \) for pure mud as a function of bed layer concentration
  \[ \tau_{ce} = \tau_{ce}(2) \]  
  \( (69) \)
- In the intermediate range \((30\% < f_2 < 50\%)\):
  \[ \tau_{ce} = f_1 \tau_{ce}(1) + f_2 \tau_{ce}(2) \]  
  \( (70) \)

Figure 6 – Mixed sediment bed structure.

The potential erosion flux depends on the composition of the bed layer. The first step is to determine a mean erosion rate per layer \( \overline{E} (\text{FLUER}_\text{LOC}) \):
- For \( f_2 < 30\% \) (sand dominant): we use the equilibrium concentration concept:
  \[ \overline{E} = \begin{cases} 
  E_1 = w_1 C_{eq}, & \text{for } \tau_0 \tau_{ce} \\
  0, & \text{otherwise} 
\end{cases} \]  
  \( (71) \)
- For \( f_2 > 50\% \) (mud dominant): the Partheniades erosion law can be applied:
  \[ \overline{E} = \begin{cases} 
  E_2 = M \left( \left( \frac{\tau_0}{\tau_{ce}} \right)^2 - 1 \right), & \text{for } \tau_0 > \tau_{ce} \\
  0, & \text{otherwise} 
\end{cases} \]
- For \( 30\% < f_2 < 50\% \) (intermediate range): the mean erosion rate is an average value of the erosion classes of both classes:
  \[ \overline{E} = f_1 E_1 + f_2 E_2 \]
17.4 Iterative procedure

The next step is to calculate the mean erosion flux for both classes of bed material. The underlying assumption is that the eroded depth is the same for both classes. This is done iteratively, by checking layer by layer (starting from the top) how much sediment there is in the layer and how much time it would take to empty that layer entirely: This procedure is necessary because the bed is stratified and the eroded depth is unknown.

At the start of time step \((t_0)\), the first layer whose thickness is non-zero is eroded, at a given rate \(e(j)\). For each erodible layer \(j\) with \(E(j) > 0\), the time interval \(dt(j)\) to erode completely layer \(j\) is estimated based on the mass of the layer:

\[
dt(j) = \frac{dM_s(j)}{\rho_s E(j)} = \frac{dE_s(j)C_s(j)}{\rho_s E(j)},
\]

where the potential erosion rate \(E(j)\) decreases and tends to zero as \(j\) increases. The last (non-empty) layer (noted \(j_{\text{max}}\)) to be eroded is obtained by emptying all successive top layers until:

\[
\sum_{j=1}^{j_{\text{max}}-1} dt(j) < Dt < \sum_{j=1}^{j_{\text{max}}} dt(j).
\]

For \(j_{\text{max}}\), the time left to erode the last layer is \(Dt_{\text{max}} = Dt - \sum_{j=1}^{j_{\text{max}}-1} Dt(j)\). The total (mud and sand) mass potentially eroded during this process is:

\[
M_s = \sum_{j=1}^{j_{\text{max}}-1} M_s(j) + \rho_s E(j_{\text{max}})Dt_{\text{max}}.
\]

The mean mass erosion flux of the sand-mud mixture \(E_s\) (Kg/m/s) is therefore estimated as

\[
E_s = \frac{M_s}{Dt}.
\]

Here we assume that both mud and sand are eroded simultaneously to the same depth, therefore the eroded volume flux of each phases is differentiated by their mass percent:

\[
E_1 = \frac{M_{s1}}{\rho_s Dt}, \quad E_2 = \frac{M_{s2}}{\rho_s Dt}.
\]

17.5 Definition scheme of sand-mud mixture in Sisyphe

17.5.1 Assumptions

In this framework, the sand particles are assumed to be fully embedded in the cohesive framework. So this is valid for cohesive dominant regime. The density of individual primary or sand particles is assumed to be constant and equal to \(\rho_s = 2650\) kg/m³.
17.5.2 Bed structure

The sand-mud mixture is characterized by the volume percent of each class:

\[ p_i = \frac{V_i}{V_t} = \frac{E_{s_i}}{E_t} \]

where \( V_i \) is the volume occupied by the sand or mud particles \( i = 1, 2 \), \( V_t = V_1 + V_2 \) is the total volume, \( E_{s_i} \) is the volume per unit surface area of phase \( i \) and corresponds to the vertical thickness of each phase (voids are not included), and \( E_t \) is the total volume (sand + mud) per unit surface area \( E_t = E_1 + E_2 \). It is also useful to define the mass percent of each class, for sand \( f_1 \) and for mud \( f_2 \):

\[ f_1 = \frac{\rho_s V_1}{M_t}; \quad f_2 = \frac{C_2 V_2}{M_t} \]

where \( C_2 \) is the mud density, which varies with distance for the bed surface and \( M_t \) the total mass of the sand-mud mixture:

\[ M_t = C_2 V_2 + \rho_s V_1 \]

Once sediment particles have been put in suspension, they are transported independently, by solving for each class a transport equation for the volume solid concentrations, defined as:

\[ C_i = \frac{V_i}{V_t} \]

with \( V_t = V_1 + V_2 + V_{water} \). The volume and density of suspended cohesive flocs vary as a result of flocculation, so what is conserved here is the volume of primary constitutive particles of density \( \rho_s \).

17.6 Suspended sediment transport equation

Both sand and mud are treated separately and independently. We solve here the volume 2D (depth integrated) concentrations of the primary particles, assuming constant density \( \rho_s \):

\[
\frac{\partial C_i}{\partial t} + U_{\text{conv}} \frac{\partial C_i}{\partial x} + V_{\text{conv}} \frac{\partial C_i}{\partial y} = \left[ \frac{\partial}{\partial x} \left( \epsilon_s \frac{\partial C_i}{\partial x} \right) + \frac{\partial}{\partial y} \left( \epsilon_s \frac{\partial C_i}{\partial y} \right) \right] + \left( \frac{E_i - D_i}{h} \right)
\]
- For sand: \( D_1 = w_s C F_1 \), with \( F_1 \) the ratio between the near bed concentration and depth-averaged concentration.
- For mud:
  \[
  D_2 = \begin{cases} 
  w_s C \left(1 - \frac{\tau}{\tau_d}\right), & \text{for } \tau \leq \tau_d \\
  0, & \text{otherwise}
  \end{cases}
  \]

with \( \tau_d \) the critical bed shear stress for deposition.

\[\text{Figure 8 – Mixed sediment in suspension.}\]

\[\textbf{Attention}\]
In the subroutines, the definitions of Sisyph variables are: horizontal point index: I; layer index: K; number of classes of bed material (NSICLA = 2); \( J = 1 \): sand; \( J = 2 \): mud; grain sizes: FDM(J) and number of bed layers: COUCH_TASS.

\[\textbf{Attention}\]
For pure mud, the bed shear strength increases as the concentration of the mud increases while consolidation is taking place. Mud concentration per layer: \( \text{CONC}_\text{VASE}(K) \) (Kg/m³); critical erosion shear stress per layer: \( \text{TOCE}_\text{VASE}(K) \) (N/m²).
Keywords

To enable the mixed sediment transport in Sisyphe, the following keywords should be specified in the steering file:

- **MIXED SEDIMENT** = YES (= NO, default option)
- **COHESIVE SEDIMENTS** = YES (= NO, default option. In Sisyphe, the first sediment class is always sand and the second is mud)
- **NUMBER OF SIZE-CLASSES OF BED MATERIAL** = 2 (NSICLA)
- **INITIAL FRACTION FOR PARTICULAR SIZE CLASS** = \( f_1 \); \( f_2 \) (example := 0.80; 0.20)
- **SEDIMENT DIAMETERS** = \( D_1 \), \( D_2 \) (for non-cohesive sediments \( D_1 \geq 6.0 \times 10^{-5} \), for cohesive sediments \( D_1 \leq 6.0 \times 10^{-5} \), example with 300 \( \mu \)m for sand and 60 \( \mu \)m for mud := 0.0003; 0.00006)
- **SEDIMENT DENSITY** = 2650 (the sediment density is only for sand, while the mud density is defined in the keyword MUD CONCENTRATION PER LAYER)
- **SETTLING VELOCITIES** = \( w_{s1} \), \( w_{s2} \) (example := 4. \( E \) \(-02; 1.\( E \) \(-02, if this one is active, only one value is taken for calculation. It is corrected later in the subroutine)
- **INITIAL SUSPENSION CONCENTRATIONS** (example := 0.0002; 0.0002)
- **NUMBER OF LAYERS OF THE CONSOLIDATION MODEL** (NOMBLAY = 10, default option)
- **MUD CONCENTRATION PER LAYER** (in kg/m\(^3\), CONC_VASE= 50.; 100.;..., default option)
- **CRITICAL EROSION SHEAR STRESS OF THE MUD** (example := 0.5; 10.0)

17.7 Fortran subroutines for mixed sediment transport

- **INIT_SEDIMENT**: This subroutine initializes the bed composition (by calling INIT_MIXTE for mixed sediment transport) and calculates the settling velocity, if not present in the steering file. Moreover, shields parameter and CRITICAL EROSION SHEAR STRESS OF THE SAND known as TOCE_SABLE is calculated in this subroutine as well.
- **INIT_MIXTE**: This subroutine calls INIT_Compo_COH to initialize the composition of the sediment bed and also check the thickness of each bed layer. The initial masses of sand and mud are also calculated here. It can be seen that XMVS is the sand density defined in the steering file while \( \text{CONC}(I,J) \) is the mud density of each bed layer defined in the steering file to known as MUD CONCENTRATION PER LAYER. For this subroutine, the density of the sand is assumed to be constant (XMVS) and the following variables are defined:
  - Mass per unit surface area of the sand : \( MS\_SABLE(I,K) \) (Kg/m\(^2\))
  - Mass per unit surface area of the mud : \( MS\_VASE(I,K) \) (Kg/m\(^2\))
  - Total Mass per layer : \( MS\_TOTAL(I,K) = MS\_SABLE(I,K) + MS\_VASE(I,K) \)
  - Mass percent of each phase per layer : \( Fs(I,K,1) = MS\_SABLE(I,K)/MS\_TOTAL(I,K) \) and \( Fs(I,K,2) = MS\_VASE(I,K)/MS\_TOTAL(I,K) \)
  - Initial mass of the sand bed : \( MASS0 \)
  - Initial mass of the mud bed : \( MASV0 \)
- **INIT_Compo_COH**: This subroutine initializes fraction distribution of sand and mud. The thickness of each bed layer is also defined here. For this simulation, there are two bed layers defined in this subroutine as the sum of the thickness of mud per layer (EPAI_VASE) and the thickness of sand per layer (EPAI_SABLE). The modification is made for reading sand fraction from data file. We do not take into account the porosity of sand, because we assume the void to be entirely filled up by the fine particles. This may be wrong when the mud percent % is lower than the void ratio (in case of bedload layer for example). Up to 6.3, the present the mixte sediment formulation is only valid assuming the sand grains to be imbedded in the cohesive frame.
  - Release 6.3: the thickness of each sand layer corresponds here to the volume occupied by the sand particles, per unit surface area.
  - Bed layer thickness for sand : \( EPAI\_SABLE(K) \) (m)
  - Bed layer thickness for mud (m) : \( EPAI\_VASE(K) \) (m)
  - Bed layer thickness (mud and sand) : \( ES(I,K) = EPAI\_SABLE(K) + EPAI\_VASE(K) \)
  - Volume percent of each phase : \( AVAIL(I,K,1) = EPAI\_SABLE(K)/ES(I,K) \) and \( AVAIL(I,K,2) = EPAI\_VASE(K)/ES(I,K) \)
  - Future release: introduce a test on AVAIL (only correct if \( AVAIL(I,K,2) > \) void ratio of \( \approx 0.4 \))
- SUSPENSION_COMPUTATION: This is the main subroutine for the computation of the suspension load and the bed elevation. It has the following steps:
  1. COMPUTES THE REFERENCE ELEVATION --> ZREF
  2. ADVECTION VELOCITY --> UCONV, VCONV, TAKING INTO ACCOUNT THE VERTICAL PROFILE OF CONCENTRATIONS AND VELOCITIES
  3. EROSION FLUX: FLUER In this part, the subroutine SUSPENSION_FLUX_MIXTE is called to compute the suspension for the sand and mud mixture.
  4. DEPOSITION FLUX: FLUDPT =WC*T2 This subroutine calls SUSPENSION_DEPOT to compute the sand and mud deposition.
  5. DIFFIN A SPECIFIC TREATMENT IS DONE IF THE ADVECTION METHOD IS THE CHARACTERISTICS: FREE OUTPUTS ARE TREATED LIKE DIRICHLET. THIS SPECIFIC TREATMENT IS CANCELLED HERE BY SENDING A MODIFIED VALUE FOR RESOL.
  6. BOUNDARY CONDITIONS: CBORCONC_V The keyword IMP_INFLOW_C is found here for controlling the calculation of equilibrium concentration at open boundaries.
  7. SOLVING TRANSPORT EQUATION IF METHOD OF CHARACTERISTICS
  8. SOURCE AND SINK TERMS (IMPLICIT SOURCE TERM FOR THE DEPOSITION T9 AND EXPLICIT SOURCE TERM WITHOUT PUNCTUAL SOURCES T11)
  9. ADVECTION-DISPERSION STEP CONFIGURATION OF ADVECTION
  10. BED EVOLUTION DUE TO NET EROSION/DEPOSITION FLUX The subroutine SUSPENSION_EVOL is called from here to calculate the bed evolution.
Références


